

STUDY OF THE PROCESS OF PENETRATION OF A ROTATING BODY INTO
THE ENVIRONMENT BEHIND THE SHOCK WAVE FRONT

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Annotation. Based on his experimental studies, the author determined the functionally dependent Lamé variables $\lambda(\varepsilon, \varepsilon_i), G(\varepsilon, \varepsilon_i)$ and used them to model a homogeneous soil mass as a continuous medium with elastic-plastic properties. Taking into account the combined effect of normal and tangential stresses, using the example of a study of the propagation of strong shock waves in homogeneous media with cylindrical symmetry, the mechanical parameters of symmetrical solids with rotational motion and initial velocity upon penetration into this medium were determined and new scientific solutions were obtained.

Key words. Elastoplastic medium, immersion of rotating bodies in the medium, explosion, normal and impact stresses, modeling, shock wave front.

Introduction.

In ancient times, scientists such as Euler, Vuich, Zabudsky, Mayevsky, Ponsel studied the patterns of interaction between solids and the medium under certain restrictions, expressing the resistance force of the medium F as the sum of three forces in determining the resistance of the obstacle and the depth of immersion [1, 2]:

$$F = c_1 v^2 + c_2 v + c_3 \quad (1)$$

where v - body speed during penetration; c_1, c_2, c_3 - constant values with a positive sign, depending mainly on the physicochemical and mechanical properties of the medium, as well as on the shape of the penetrated body.

Research combining theory and practice, aimed at the efficient utilization of energy generated by directed and purposeful powerful explosions, has remained relevant since the last century. This research encompasses processes such as excavation and soil transportation in engineering practice, the construction of underground structures, the creation of cavities and gas storage facilities, mining operations, ensuring seismic safety and stability of dams and land reclamation and hydraulic structures, as well as the safe landing of aircraft.

An introduction to the science of the theories of academician Kh.A.Rakhmatulin, including "Rakhmatulin waves" and "layer motion," as well as the model of an ideal "plastic gas," is of great importance for engineering practice. This has become possible due to the research conducted by Kh.A.Rakhmatulin, S.S.Grigoryan, A.Ya.Sagomonyan, S.S.Davydov, G.M.Lyakhov, S.S.Vylov, N.A.Tsytoivich, Jacques de Marre, G.I.Pokrovsky, A.Tate, N.V.Maevsky, N.A.Zabudsky, Yu.V.Khaydin, V.A.Veldanov, V.V.Balandin, V.A.Koronatov, V.N.Aptukov, I.V.Khromov, K.S.Sultanov, B.M.Mardonov and A.N.Nabiev. These studies encompassed the investigation of the laws governing the propagation of shock waves, as well as the theoretical modeling and experimental research of the process of penetration of rotating indenter-type bodies into deformable media. The results of these works have found wide application in engineering practice [1,2,3,4,5,6,7,10,12,14].

The mechanical-mathematical essence of the problem.

An indenter in the form of a rigid body with an initial velocity $v_0 \neq 0$ rotates around its axis of symmetry with an angular velocity $n \neq 0$ [rpm] and penetrates into soil or rock masses modeled as an elastic-plastic medium (Fig. 1).

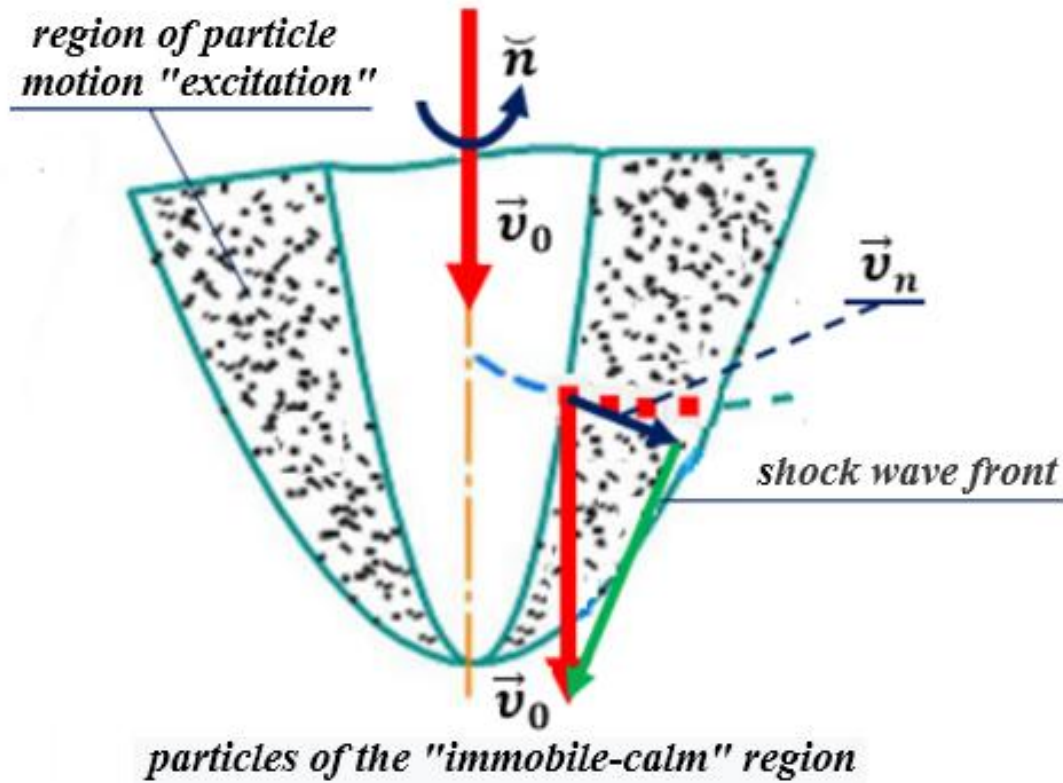


Fig. 1. Penetration of a rotating body into a medium with initial velocity

For mechanical and mathematical analysis of the process of immersion of rotating bodies in the presence of an angular speed of rotation around an axis of symmetry other than zero ($n \neq 0$ rpm), and an initial vertical speed also other than zero ($v_0 \neq 0$), it is necessary to find solutions to problems related to the law of immersion, such as determining the depth of immersion, speed and acceleration.

Typically, in a general case, the solution to such problems is reduced to the investigation and solution of one-dimensional shock waves in elastic-plastic media with cylindrical symmetry from a mechanical-mathematical point of view [1,2,9,10,11].

In the article, in accordance with the deformation theory of "Finite elastic-plastic media", using the results of experiments carried out directly by the author, the motion of indenter-like bodies rotating at a speed of n [rev/sec] (or $N \neq 0$) with the help of a special mechanical drive is studied, for a medium with a coefficient of adhesion of particles $k=0$, a coefficient of sliding friction $\mu_0 \neq 0$, an angle of internal friction of the medium $\vartheta \neq 0$ ($\mu = \sin \vartheta$, $\tau_0 = 2k \cos \vartheta$) penetrating into media where there is no internal friction.

Methods.

Before solving the problem, taking into account the physical and mechanical nature of the medium and its homogeneity, we rely on the following limitations (Fig. 2):

- the law of penetration of rotating bodies is expressed through $H(t)$, with the onset of the penetration process at any moment in time t , the immersing body acquires a velocity $\dot{H}(t)$, and its upper part is also completely located at a depth of $H(t)$;
- the inner boundary of the motion field is a circle of radius $H(t)$, which completely describes the line of intersection of the motion of the immersing body with the plane of the medium;
- Only when shock waves occur does the density of the medium change, and this is determined by the intensity of the wave's motion;
- At the front of shock waves created by impacts, only a stressed state occurs in the media;
- Shock waves propagate in the media at a velocity exceeding the plastic wave velocity.

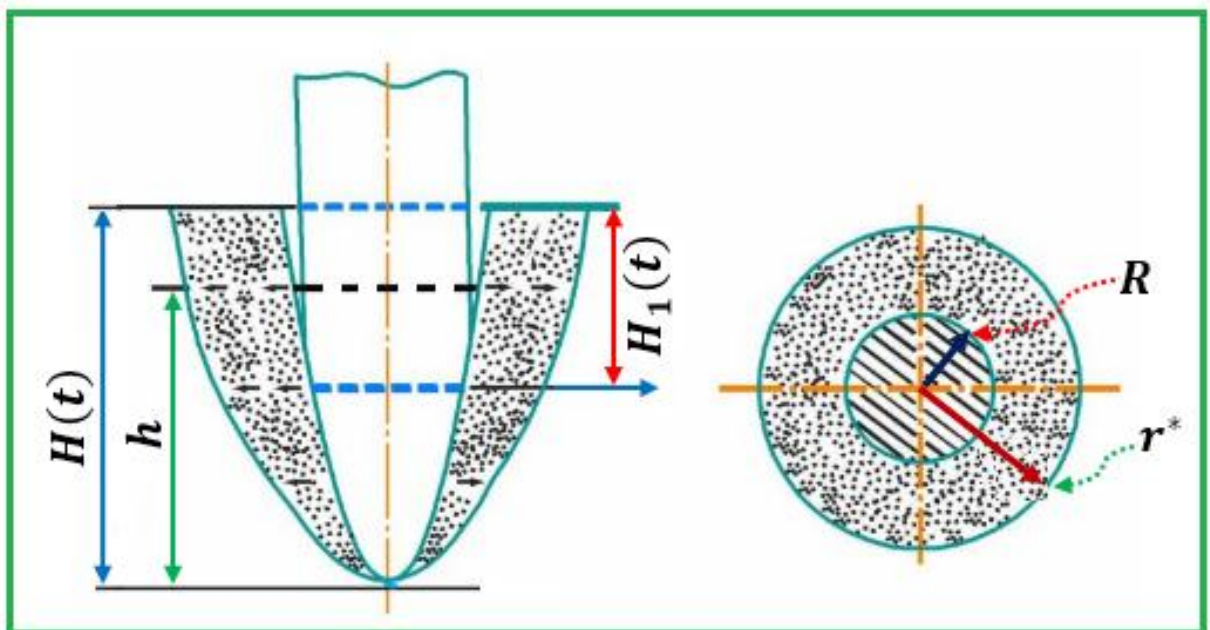


Fig.2. Description of the laws of immersion of rotating bodies

An important practical aspect of these limitations is that the density of the medium behind the shock wave is a continuous function of the timeless Lagrangian coordinate r . This allows us to conclude that the density of the medium is a continuous function of the timeless Lagrangian coordinate r . In practice, applying this limitation, complex processes directly related to shock wave formation have been studied in detail, such as the problems of flat, cylindrical, and spherical expansion of elastic-plastic media [1, 8].

An analysis of studies devoted to problems and their solutions similar to those described above shows that the process of solid body penetration into media can be divided into two groups based on the nature of the study:

- Analysis of the stress-strain state of the medium subjected to impact (this determines the maximum force acting from the medium on the immersed body).
- Calculation of the immersion depth (this determines the speed and acceleration of immersion).

Based on this, in order to study the laws governing the penetration of cone-shaped solids into elastic-plastic media, we will consider the mechanism for determining the forces acting on a submerged solid from the medium.

In the theory of elasticity and plasticity, when the orientation of the faces of a selected element changes, the stresses on its surface change, that is, increase or decrease.

In general, it is always possible to find an orientation for an element isolated from a medium such that, in a stress-strain state, in addition to normal stresses, tangential stress components also act on its faces (Fig. 3, a, b).

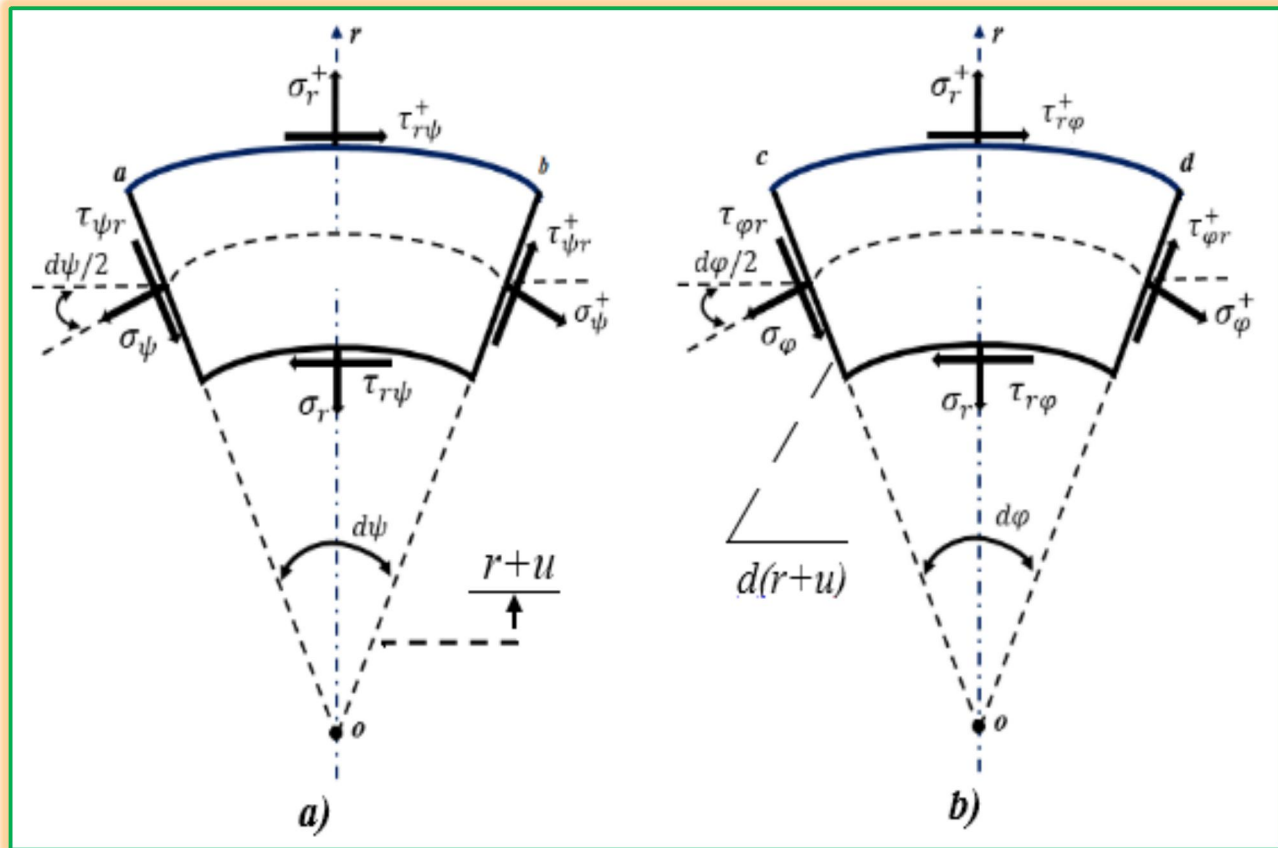


Fig. 3. The influence of stress on an element mentally isolated from the environment

Naturally, both elements describe the same stress state: the outer element is characterized by principal stresses, while the inner element is characterized by stresses on inclined sections.

In turn, from the "inner element," one can distinguish an element – a particle whose faces are subject only to normal stresses, serving as principal surfaces.

The theory of elasticity and plasticity substantiates that such elements can be selected in any (infinite) number. Moreover, not only all of them, but each of them describes a specific stress state at a given point in the medium.

The stress state of the medium is volumetric, while the deformation state is uniaxial (radial), and the influence of the normal stress σ_r in the radial direction is strong.

In gas and wave dynamics, it has been established that if the states of a medium are defined by a single spatial coordinate, then the gas motion is called one-dimensional.

Typically, gas motion can be considered close to a single dimension in the following cases:

- movement behind and in front of a piston in an adiabatic compression tube;
- gas motion in a shock tube;
- movement during plane, cylindrical, and spherical detonation explosions;

- gas motion at a certain distance from the center of an atomic explosion, etc.

Taking into account the combined approach to the problem from the mechanical and mathematical points of view, the Prandtl plasticity conditions, the combined action of normal and shear stresses, as well as their invariance, an equation for one-dimensional dynamic soil movement in Lagrange variables in the processes of a strong explosion with cylindrical symmetry was obtained in a more generalized form [8, 9, 10, 11, 12]:

$$\rho_0 r \cdot \frac{\partial^2 u}{\partial t^2} = (r+u) \cdot \frac{\partial \sigma_r}{\partial r} - 2 \cdot \tau_0 \cdot \cos 2\varphi \cdot \frac{\partial}{\partial r} (r+u) \quad (2)$$

The continuity equation for particles of a medium in Lagrange variables, first proposed by Academician Kh.A.Rakhmatullin in the study of the spatial motion of media and applied for the systemic solution of engineering problems, has the following form:

$$\frac{1}{2} \cdot \frac{\partial}{\partial r} (r+u)^2 = \frac{\rho_0}{\rho(r)} \cdot r \quad (3)$$

In general, to solve these problems from a mathematical point of view, it is necessary to integrate a partial differential equation, which can represent the motion of media.

However, the above system of equations is not closed, since it contains four unknown functions: the stresses σ_r and σ_φ , the displacement u , and the density of the medium ρ . To close the system of equations, one can practically use the generalized functional Lamé coefficients obtained as a result of experimental studies of the laws of static and dynamic resistance of media, based on the constraint “Existence of elastic potential $U(\varepsilon, \varepsilon_i)$ ” [1, 2, 9, 10, 12]:

$$G(\varepsilon, \varepsilon_i) = \frac{1}{3\varepsilon_i} \cdot \sigma_i(\varepsilon, \varepsilon_i); \quad \lambda(\varepsilon, \varepsilon_i) = \frac{1}{3\varepsilon} \cdot \sigma(\varepsilon) - \frac{2}{9} \cdot \frac{1}{\varepsilon_i} \cdot \sigma_i(\varepsilon, \varepsilon_i) \quad (4)$$

In accordance with the law of conservation of mass and the theorem on the change in momentum, taking into account the boundary conditions and the continuity equation drawn up for a front with a strong shock wave, as well as the fact that the motion begins from the coordinate $r_0=0$, the possibility of transforming the equation of motion with parameters characterizing the internal friction of the medium $\mu=9=0$ is substantiated as follows [2, 8, 14]:

$$-\sigma_r = \rho_0 (\dot{R}^2 + R\ddot{R}) \int_0^{r^*} \frac{r dr}{(2\psi(r) + R^2)} - \rho_0 (R\dot{R})^2 \cdot \int_0^{r^*} \frac{r dr}{(2\psi(r) + R^2)^2} + \frac{\rho_0}{1-b(r^*)} \cdot \frac{(R\dot{R})^2}{r^{*2}} + 2\tau_0 \cos 2\varphi \left\{ \ln \frac{r^*}{R} \right\} + p_\alpha + \left[\frac{\sigma(\varepsilon^*)}{\varepsilon^*} + \frac{4}{9} \cdot \frac{\sigma_i(\varepsilon^*, \varepsilon_i^*)}{\varepsilon_i^*} \right] \varepsilon_i^* \quad (5)$$

Typically, for the case of cylindrical symmetry, firstly, the following relationships hold between the density of the medium, the radial displacement, the relative deformation and the intensity of deformation at the front of strong shock waves:

$$\varepsilon^* = 1 - \frac{\rho_0}{\rho^*} = u_r^*; \quad \varepsilon^* = 1 - b(\rho^*); \quad \varepsilon_i^* = \frac{2}{3} \cdot \varepsilon^* = \frac{2}{3} \cdot \left(1 - \frac{\rho_0}{\rho^*} \right) \quad (a)$$

Secondly, the Lagrange coordinate function is expressed as follows:

$$\psi(r) = \int_{r_0}^r \frac{\rho_0 r}{\rho(r)} dr \quad \text{and} \quad \psi(r^*) = \int_{r_0}^{r^*} \frac{\rho_0 r}{\rho(r)} dr \quad (b)$$

According to the law of parity of tangential stresses and the notation $\mu_\beta = (1 - \mu_0 \tan \beta)^{-1}$, the normal σ_β and tangential τ_β stresses on the inclined surfaces of a particle isolated from a medium subjected to a stress-strain state under the influence of an immersing body are equal to [14]:

$$\sigma_\beta = \mu_\beta \left(\sigma_r + \frac{C_0 N}{n} \tan \beta \right); \quad \tau_\beta = \mu_0 \mu_\beta \left(\sigma_r + \frac{C_0 N}{n} \tan \beta \right) \quad (6)$$

Here $N = N_{kvt}$ is the power expended on the movement of a penetrating body of circular cross-section with an external diameter D at a rotational speed n [rpm]; $C_0 \approx 48657,33 \cdot D^{-3}$ is a coefficient depending on the diameter of the penetrating body; β is the angle between the z -axis of symmetry of the particle and the tangent to the generatrix of the penetrating body; μ_0 is the coefficient of sliding friction (in practice it is chosen equal to $\mu_0 \approx 0,2 \div 0,25$).

In the particular case where $N=0$, the results obtained are those given in [1,2]:

$$\sigma_\beta = \mu_\beta \sigma_r; \quad \tau_\beta = \mu_0 \mu_\beta \sigma_r \quad (c)$$

Naturally, certain forces act on an element mentally isolated from the medium,

$ds = 2\pi f(x) \cdot \sqrt{1+f^2(x)} \cdot dx$ in the normal and tangential directions. Based on this, the following expressions can be formulated:

$$dF = -\sigma_\beta \cdot ds \cdot \sin \beta; \quad dQ = -\tau_\beta \cdot ds \cdot \cos \beta \quad (d)$$

Then we get the following relationship:

$$dF + dQ = -2\pi \mu_\beta (\sin \beta + \mu_0 \cos \beta) \left(\sigma_r - \frac{C_0 N}{n} \tan \beta \right) f(x) \sqrt{1+f^2(x)} dx \quad (7)$$

The equation of the generatrix ("oral cavity") of a body penetrating into a medium, $R=f(x)$ at a penetration time $t \geq t_1$ is equal to:

$$R_1 = f(x) = f[H(t) - H_1(t_1)] \quad (k)$$

In processes of strong explosions or when penetrating bodies act on media without internal friction at high speeds, the normal stress in the radial direction is governed by the relation $\sigma_r \approx -p + p_\alpha$ (where p is the pressure in the section $H_1(t_1)$, mentally separated from the medium, and $p_\alpha \approx 1 \text{ kg/cm}^2$ is the atmospheric pressure).

After appropriate simplifications, (7) can be written as follows:

$$dF + dQ = 2\pi \frac{\mu_0 + f(x)}{1 - \mu_0 f(x)} \cdot \left[(p - p_\alpha) - \frac{C_0 N}{n} f(x) \right] f(x) dx \quad (8)$$

For the case where the indenter bodies, turning into a cylinder at a height h , penetrate the medium with an opening angle of 2β , the following relationships can be written:

$$R_1 = \tan \beta x = \tan \beta [H(t) - H_1(t_1)], \quad \dot{R}_1 = \tan \beta \dot{H}, \quad \ddot{R}_1 = \tan \beta \ddot{H} \quad (9)$$

According to the above reasoning and the notation $a = (1 - b_1)^{-1}$, formulas (8) and (5) are expressed as follows:

$$F + Q = 2\pi \tan^2 \beta \frac{1 + \mu_0 \cdot c \tan \beta}{1 - \mu_0 \tan \beta} \int_0^H \left[(p - p_\alpha) - \frac{C_0 N}{n} \tan \beta \right] x dx \quad (10)$$

$$p - p_\alpha = \frac{\rho_0}{2b_1} \ln a \cdot f(x) \cdot \dot{f}(x) \cdot \dot{H} + \frac{\rho_0}{2b_1} \left\{ \ln a \left[f(x) \cdot f'(x) + f^2(x) \right] + b_1 f^2(x) \right\} \cdot \dot{H}^2 + \tau_0 \cos 2\varphi \ln a + \frac{2}{3} \left\{ \sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i \left[(a^{-1}) \right] \right\}$$

Let us determine the pressure from the last formula using the relations (9):

$$p - p_\alpha = \frac{\rho_0}{2b_1} \ln a \cdot \tan^2 \beta \cdot x \cdot \dot{H} + \frac{\rho_0}{2b_1} (\ln a + b_1) \tan^2 \beta \cdot \dot{H}^2 + 2\tau_0 \cos 2\varphi \frac{1}{2} \ln a + \frac{2}{3} \left\{ \sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i \left[(a^{-1}), \frac{2}{3} (a^{-1}) \right] \right\} \quad (11, a)$$

From mechanics it is known that the equation of dynamic motion of a body of mass m is written as follows:

$$m\ddot{H} = -(F+Q) \quad (12)$$

Having introduced the following notation $X = \mu_\beta(1 + \mu_0 \cdot c \tan \beta)$, taking into account expression (10) and the relation $C_0 N n^{-1} = \text{const}$, after appropriate simplifications we reformulate equation (12) for the immersion of indenter bodies rotating with the initial velocity into the studied medium:

$$m\ddot{H} + \frac{\pi\rho_0 \tan^4 \beta \ln a}{3b_1} X H^3 \dot{H} + \frac{\pi\rho_0 \tan^4 \beta}{2b_1} (\ln a + b_1) X H^2 \dot{H}^2 = -\pi \tan^2 \beta X \left\{ \tau_0 \cos 2\varphi \ln a + \frac{2}{3} \left\{ \sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i \left[(a^{-1}), \frac{2}{3} (a^{-1}) \right] \right\} \right\} \cdot \frac{2}{3} (a^{-1})$$

Now, taking into account the relations $\dot{H} = y'/2$, $H^2 = y$, we introduce the following notations:

$$\omega = \frac{\pi\rho_0 \tan^4 \beta \ln a}{3mb_1}; \quad \alpha = \frac{\pi\rho_0 \tan^4 \beta}{b_1} (\ln a + b_1); \quad \frac{\alpha}{\omega} = 3 \left(1 + \frac{b_1}{\ln a} \right)$$

$$c = \frac{2\pi \tan^2 \beta}{m} \left\{ \tau_0 \cos 2\varphi \ln a + \frac{2}{3} \left\{ \sigma(a^{-1}) + \frac{2}{3} \cdot \sigma_i \left[(a^{-1}), \frac{2}{3} (a^{-1}) \right] \right\} \right\} - \frac{C_0 N}{n} \tan \beta$$

Accordingly, the following first-order linear differential equation is obtained:

$$y' + \frac{X\alpha H^2}{1+X\omega H^3} \cdot y = - \frac{XcH^2}{1+X\omega H^3} \quad (13)$$

The solution to this equation is as follows:

$$y = - \left[\frac{c}{\alpha} \cdot (1+X\omega H^3)^{\frac{\alpha}{3\omega} + k_0} \right] \cdot \frac{1}{(1+X\omega H^3)^{\frac{\alpha}{3\omega}}} \quad (14)$$

From the initial condition of the problem it is easy to determine k_0 :

$$H=0: \quad y_0 = H_0^2 = v_0^2 \quad \text{or} \quad v_0^2 = k_0 - \frac{c}{\alpha}; \quad k_0 = v_0^2 + \frac{c}{\alpha} \quad (15)$$

As a result, the velocity \dot{H} and acceleration \ddot{H} of the penetrating body are easily calculated using the following formulas:

$$y = H^2 = \frac{v_0^2 + \frac{c}{\alpha}}{(1+X\omega H^3)^{\frac{\alpha}{3\omega}}} - \frac{c}{\alpha} \quad \text{and} \quad \ddot{H} = - \frac{\alpha X}{2} \cdot \frac{\left(v_0^2 + \frac{c}{\alpha} \right) \cdot H^2}{(1+X\omega H^3)^{\frac{\alpha}{3\omega} + 1}} \quad (16)$$

Using the formulas provided above, it is possible to determine the velocity \dot{H} and acceleration \ddot{H} of bodies moving in a medium with or without translational motion for the following specific cases:

1. $\mu_0 = 0$ ($X=1$) - case of absence of slipping friction force;
2. $\mu_0 = 0$ ($X=1$) - case of absence of slip friction force and $\tau_0 = 0$ - internal friction:
 - a) the case considering the constraint "on the existence of the elastic potential $U(\varepsilon, \varepsilon_i)$ ";
 - b) the case without considering the constraint "on the existence of the elastic potential $U(\varepsilon, \varepsilon_i)$ ".

Let us assume that $\mu_0 = 0$ ($X=1$) and $\tau_0 = 0$, and according to the restriction "On the existence of the elastic potential $U(\varepsilon, \varepsilon_i)$ ", we have:

$$\dot{H} = \frac{v_0}{(1+\omega H^3)^{\frac{\alpha}{6\omega}}} \quad \text{and} \quad \ddot{H} = - \frac{\alpha}{2} \cdot \frac{v_0^2 \cdot H^2}{(1+\omega H^3)^{\frac{\alpha}{3\omega} + 1}} \quad (16, a)$$

Indeed, the last two formulas correspond to the mechanical-mathematical meaning of the problem when $H \leq x_1$ (where x_1 is the "bifurcation point").

For the numerical analysis of the process of immersing rotating bodies in the soil mass, the corresponding mechanical parameters were calculated using formula (16) based on the following data, and the results are presented graphically [Fig. 4, 5]:

$$N_{dv}=75.5 \text{ kvt}, \quad n=300 \frac{\text{aln}}{\text{min}}, \quad D=0.25 \text{ m}, \quad v_0=600 \frac{\text{m}}{\text{sek}}, \quad h=0.13 \text{ m},$$

$$\vartheta=\frac{\pi}{9}, \quad \rho_0=152.9 \cdot 10^{-8} \frac{\text{g} \cdot \text{sek}^2}{\text{sm}^4}, \quad mg=10 \text{ kg}, \quad \mu=\sin\vartheta=0.342,$$

$$\tau_0=0.94 \frac{\text{kg}}{\text{sm}^2}, \quad \beta=\frac{\pi}{6}, \quad k_1=2.683 \cdot 10^3 \frac{\text{kg}}{\text{sm}^2}, \quad k_2=-4.991 \cdot 10^3 \frac{\text{kg}}{\text{sm}^2},$$

$$k_3=-5.744 \cdot 10^3 \frac{\text{kg}}{\text{sm}^2}, \quad k_4=2.498 \cdot 10^3 \frac{\text{kg}}{\text{sm}^2}, \quad \mu_0=0,2.$$

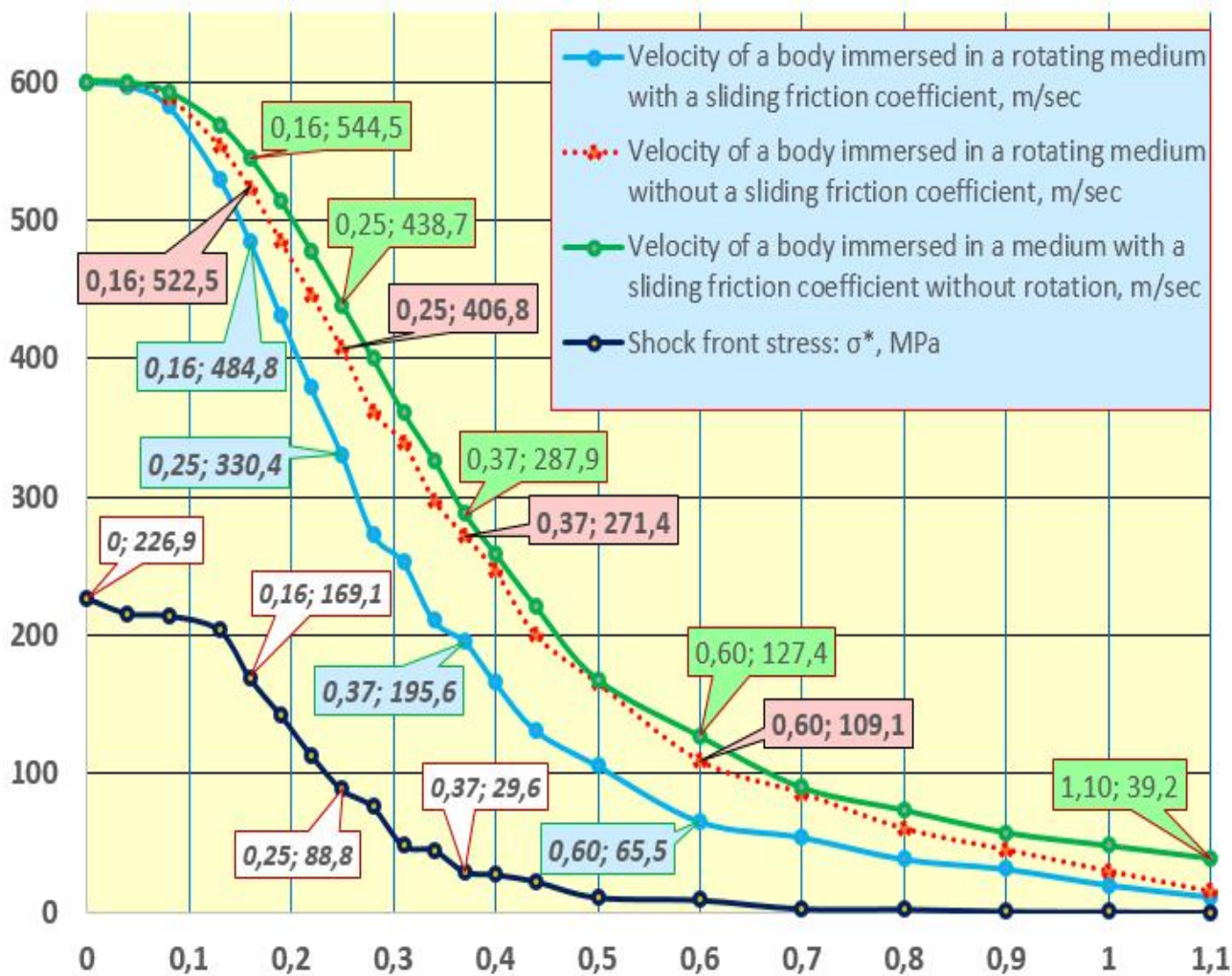


Fig.4. Patterns of change in the immersion velocity (m/s) and stresses at the shock front with immersion depth (m)

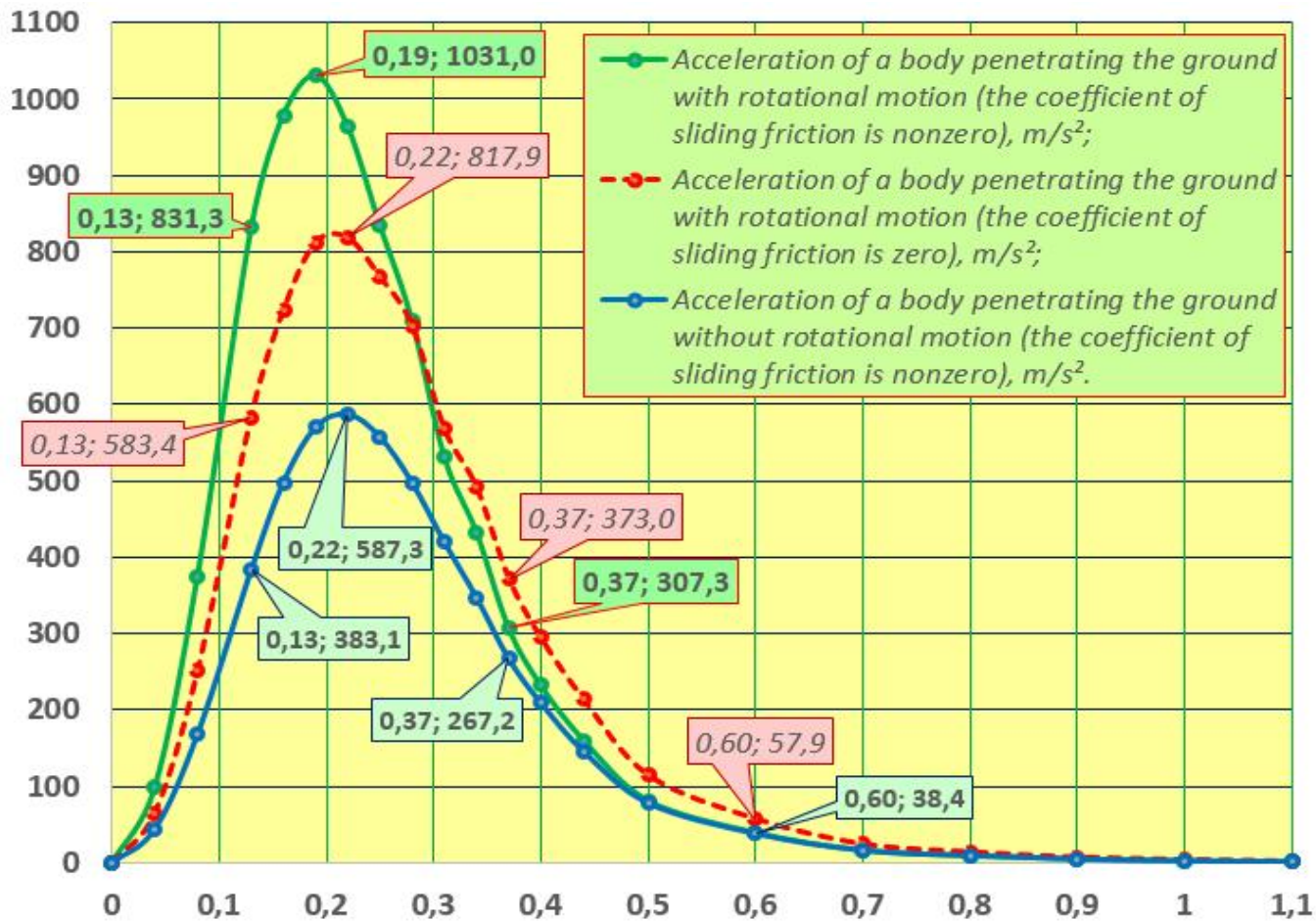


Fig. 5. Illustration of the relationship between the acceleration of rotating bodies immersed in the ground (m/s^2) and the depth of immersion (m)

During the penetration process, the values of velocity and acceleration of penetrating bodies were compared with the numerical and graphical solutions presented in [1, 2].

KEY CONCLUSIONS

The following scientific conclusions and solutions were reached during the study:

1. Based on the theory of "finite elastic-plastic deformation", the study of the laws of immersion of solid bodies rotating around an axis of symmetry with an initial velocity into a medium is used to study the propagation of shock waves in cylindrically symmetric media.

2. Based on the author's experimental research to determine the main mechanical properties of soils under the influence of high static and dynamic stresses, the Lamé coefficients in the functional relationship $[\lambda(\varepsilon, \varepsilon_i), G(\varepsilon, \varepsilon_i)]$ were improved in form and content and used to model soil masses as elastic-plastic continuous media.

3. By replacing the elastic constants in the model of a linearly deformable medium with functional experimental coefficients of stress tensors $[\lambda(\varepsilon, \varepsilon_i), G(\varepsilon, \varepsilon_i)]$, firstly, minor factors of soil deformation were taken into account, and secondly, the rotational motion of immersing bodies and the shear stress in the medium were taken into account, and on the basis of the restriction on the "existence of an elastic potential $U(\varepsilon, \varepsilon_i)$ ", a dynamic equation of motion was

derived for the first time, describing the process of immersion from a mechanical and mathematical point of view.

4. To solve the differential equations associated with the immersion process, for the first time, when forming the boundary parameters at the shock wave front, a restriction on the existence of an elastic potential $U(\varepsilon, \varepsilon_i)$ was used, taking into account that the medium behind strong shock fronts is only in a stressed state.

5. When comparing the results of this study with the scientific conclusions and solutions obtained as a result of existing studies [1, 2, 4, 5, 6, 7, 14], it was confirmed, firstly, that the patterns of speed and acceleration of a sinking body and the depth of immersion, shown in the graphs, correspond in form and content, and, secondly, it was established that in cases where the speed of rotation around the axis of symmetry $n \neq 0$ and the initial vertical speed $v_0 \neq 0$, the depth of immersion is approximately 3 times greater than in cases where $n=0$ and $v_0 \neq 0$.

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