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CONFIDENCE INTERVAL FOR THE STANDARD DEVIATION OF A NORMAL DISTRIBUTION.

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To construct a confidence interval for a single unknown population mean, μ, where the population standard deviation is known, we need x¯ as an estimate for μ, and we need the margin of error. Here, the margin of error is called the error bound for a population mean (EBM). The sample mean, x¯, is the point estimate of the unknown population mean, μ.

The confidence interval (CI) estimate will have the form (point estimate – error bound, point estimate + error bound) or, in symbols, (x¯–EBM, x¯+EBM).

The margin of error (EBM) depends on the confidence level (CL). The confidence level is often considered the probability that the calculated confidence interval estimate will contain the true population parameter. However, it is more accurate to state that the confidence level is the percentage of confidence intervals that contain the true population parameter when repeated samples are taken. Most often, the person constructing the confidence interval will choose a confidencelevel of 90 percent or higher, because that person wants to be reasonably certain of his or her conclusions.

Another probability, which is called alpha (α), is related to the confidence level, CL. Alpha is the probability that the confidence interval does not contain the unknown population parameter. Mathematically, alpha can be computed as α=1−CL.

Example.

- Suppose we have collected data from a sample. We know the sample mean, but we do not know the mean for the entire population.
- The sample mean is seven, and the error bound for the mean is 2.5.

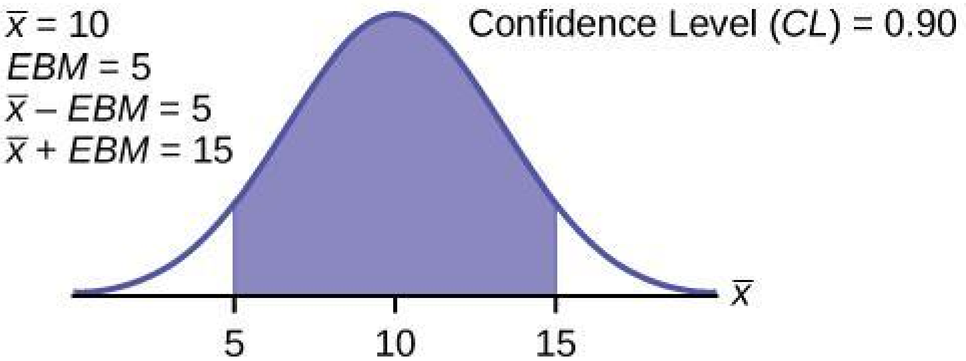
x¯ and EBM = 2.5.

The confidence interval is (7 – 2.5, 7 + 2.5), and calculating the values gives (4.5, 9.5).

If the confidence level is 95 percent, then we say, "We estimate with 95 percent confidence that the true value of the population mean is between 4.5 and 9.5."

A confidence interval for a population mean with a known standard deviation is based on the fact that the sample means follow an approximately normal distribution. Suppose that our sample has a mean of x¯ = 10, and we have constructed the 90 percent confidence interval (5, 15) where EBM = 5.

To get a 90 percent confidence interval, we must include the central 90 percent of the probability of the normal distribution. If we include the central 90 percent, we leave out a total of α = 10 percent in both tails, or 5 percent in each tail, of the normal distribution.



The critical value 1.645 is the z-score in a standard normal probability distribution that puts an area of 0.90 in the center, an area of 0.05 in the far left tail, and an area of 0.05 in the far right tail. To capture the central 90 percent, we must go out 1.645 standard deviations on either side of the calculated sample mean. The critical value will change depending on the confidence level of the interval.

It is important that the standard deviation used be appropriate for the parameter we are estimating, so in this section, we need to use the standard deviation that applies to sample means, which is σn√. The fraction σn√ is commonly called the standard error of the mean in order to distinguish clearly the standard deviation for a mean from the population standard deviation, σ.

In summary, as a result of the central limit theorem, the following statements apply:

- X¯¯¯¯¯ is normally distributed, that is, X¯¯¯¯¯ ~ N(μX,σn√).
- When the population standard deviation σ is known, we use a normal distribution to calculate the error bound.

Calculating the Confidence Interval

To construct a confidence interval estimate for an unknown population mean, we need data from a random sample. The steps to construct and interpret the confidence interval are as follows:

- Calculate the sample mean, x¯, from the sample data. Remember, in this section, we already know the population standard deviation, σ.
- Find the z-score that corresponds to the confidence level.
- Calculate the error bound EBM.
- Construct the confidence interval.

- If we denote the critical z-score by za2, and the sample size by n, then the formula for the confidence interval with confidence level Cl=1−α, is given by (x¯−za2×σn√,x¯+za2×σn√).
- Write a sentence that interprets the estimate in the context of the situation in the problem. (Explain what the confidence interval means, in the words of the problem.)

We will first examine each step in more detail and then illustrate the process with some examples.

Finding the z-Score for the Stated Confidence Level

When we know the population standard deviation, σ, we use a standard normal distribution to calculate the error bound EBM and construct the confidence interval. We need to find the value of z that puts an area equal to the confidence level (in decimal form) in the middle of the standard normal distribution Z ~ N(0, 1).

The confidence level, CL, is the area in the middle of the standard normal distribution. CL = 1 – α, so α is the area that is split equally between the two tails. Each of the tails contains an area equal to α2.

The z-score that has an area to the right of α2 is denoted by zα2.

For example, when CL = 0.95, α = 0.05, and α2 = 0.025, we write zα2 = $z_{0.025}$.

The area to the right of $z_{0.025}$ is 0.025 and the area to the left of $z_{0.025}$ is 1 – 0.025 = 0.975.

zα2 = z0.025 = 1.96, using a calculator, computer, or standard normal probability table.

Normal table (see appendices) shows that the probability for 0 to 1.96 is 0.47500, and so the probability to the right tail of the critical value 1.96 is 0.5 – 0.475 = 0.025.

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