

**VIBRATION MANAGEMENT OF DYNAMIC MUFFLERS IN MECHANICAL SYSTEMS WITH A FINITE NUMBER OF DEGREES OF FREEDOM**

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**Annotation**

In this research, a method for the dynamic synthesis of vibration dampers for a viscoelastic mechanical system with a finite number of degrees of freedom is presented. The considered system consists of several bodies (in particular, two) that are elastically connected to the protected object. The study is focused on the development of an effective vibration protection system for electronic devices operating under external dynamic disturbances, especially in the vicinity of resonant frequencies. In the process of designing such a system, the principles of automatic control theory together with mathematical modeling methods are applied.

The proposed approach is based on the feedback concept, which allows improving the efficiency of vibration suppression. An algorithm for generating feedback signals within an information-measuring system for active vibration protection of electronic devices is developed. The main feature of this algorithm is the consideration of the phase difference between external excitation and the additional compensating vibration effects. On the basis of the proposed mathematical model, feedback signals can be generated for each control channel, which makes it possible to reduce the overall vibration impact acting on the electronic equipment.

**Introduction**

The continuous improvement of modern machinery is associated with increasing productivity, higher operating speeds of working elements, reduced material consumption, and greater mechanical loads caused by vibration and shock effects. At the same time, ensuring the reliability of equipment and maintaining safe operating conditions have become essential engineering requirements. These factors significantly increase the importance of solving vibration protection problems, which today represent one of the key research directions in the field of modern machine dynamics. Contemporary machines are often equipped with complex automatic control systems, which create opportunities to employ external energy sources for monitoring and regulating the dynamic behavior of technical objects. In this context, maintaining an acceptable level of vibrational motion can be considered an important technological quality control problem. Fundamental results related to these issues have been presented in numerous scientific studies [1–6].

Among the existing approaches, active vibration absorbers are considered one of the most effective solutions. In addition to traditional damping elements, such systems incorporate components supplied by an additional energy source, which makes it possible to vary the stiffness of suspension elements and thereby reduce the influence of vibration loads on radio-electronic equipment (REU). These devices are designed to decrease vibration amplitudes not only at resonant frequencies but also across a specified frequency range. However, the introduction of additional vibration measurement devices and control elements significantly complicates their structural design. For this reason, such vibration protection systems are typically used only in special cases involving highly responsible or sensitive electronic equipment. Moreover, each vibration absorber must be tuned to a particular operating frequency—usually corresponding to the first natural frequency of the device. Since this value

may vary within a certain range for different products, the integration of adaptive mechanisms into vibration protection systems becomes an important requirement [7–9].

A distinctive characteristic of many mechanical systems, when compared with traditional control objects in automatic control theory, is the close correspondence between their physical and informational representations. At a certain level of analysis, mechanical systems can be interpreted in terms of transformations of power or kinematic parameters that characterize the dynamic state of the system. This interpretation follows naturally from the fundamental laws of mechanics and the accepted conceptual framework used for analyzing mechanical motion. The self-organization of relative motion under external disturbances, as well as the response of systems to different types of dynamic influences, is closely related to the implementation of various feedback mechanisms.

The present study reflects the results of investigations aimed at developing methods for evaluating possible dynamic modes of oscillation damping in mechanical systems based on the feedback principle. In practical applications, vibration protection is often required primarily near resonant frequencies, since vibrations at other frequencies usually have a much smaller impact on radio-electronic equipment. Therefore, the development of a structurally simple yet effective information–measurement system for controlling active vibration protection of REU represents an actual and practically significant task. Such a system should provide efficient reduction of vibration loads, particularly in the resonance region.

With the rapid progress of modern technology, the problem of protecting devices—such as those used in aircraft and other moving objects—from harmful vibration and shock effects is becoming increasingly critical. This situation is largely explained by two interrelated trends. On the one hand, the power of machines and power installations per unit weight continues to grow, which increases the proportion of energy dissipated in the form of vibration and noise. On the other hand, the sensitivity of modern devices and the complexity of precision mechanisms are constantly increasing, leading to stricter requirements for their reliability and uninterrupted operation. Vibrational influences on device components, including sensitive objects such as electronic or biological systems, may lead to nonlinear deformation phenomena depending on the physicomachanical properties of the materials involved. To address these challenges, linearly active vibration protection systems have recently been widely applied. In such systems, control forces are directly applied to the protected object in order to compensate for vibration disturbances. Nevertheless, most existing systems demonstrate high efficiency only in compensating narrow-band vibration processes. When broadband vibration disturbances must be suppressed, additional difficulties arise related to maintaining system stability, achieving the required performance quality, and preventing nonlinear effects during vibration processes.

**Purpose of work** The main objective of this research is to develop methods of mathematical and computer modeling for analyzing the dynamics of both active and passive vibration protection systems. These methods are intended to improve and evaluate the effectiveness of such systems when applied to nonlinear mechanical systems. In addition, the study aims to enhance the efficiency of passive and active vibration protection of various technical objects, including electronic devices, through the optimal selection of system parameters while taking into account the structural dissipative characteristics of the system.

To accomplish this objective, the following task is addressed: the development of a mathematical and computational model that allows the simultaneous analysis and application of active and passive vibration protection dynamics, thereby improving the overall efficiency of their implementation in mechanical systems.

**Statement of the Problem and Methods of Solution**

In this study, the dynamic characteristics of the stress–strain state of a dissipative mechanical system composed of both deformable and rigid bodies are analyzed. By assuming that the motion of the system has an oscillatory character, it becomes possible to apply the so-called freezing procedure [10]. The application of this approach leads to the formulation of complex physical relationships describing the behavior of deformable elements with negligible (zero) volume.

$$F_e = -c_e \Delta e = -c_e [1 - \Gamma_e^c(\omega_R) - i\Gamma_e^s(\omega_R)] \Delta e. \quad (1)$$

$$\Gamma_{\lambda,m}^c(\omega) = \int_0^{\lambda} R_{\lambda,m}(\tau) \cos \omega \tau d\tau \quad ;$$

$$\Gamma_{\lambda,\mu}^s(\omega) = \int_0^{\lambda} R_{\lambda,\mu}(\tau) \sin \omega \tau d\tau$$

$F_e$  – effort in  $i$ -om concentrated element,  $\Delta e$  -elongation of this element.

Here  $E$ - instant modulus,  $A$ ,  $\alpha$  and  $\beta$  -dimensionless parameters. The parameters of the relaxation core and the instantaneous elastic modulus are determined from quasistatic experiments by the technique described in [11]. When stating the problem of the natural and forced oscillations of the system, the principle of possible displacements is used, according to which the sum of all active forces acting on the system, including inertia forces, is zero:

$$\delta A = \delta A_\sigma + \delta A_u + \delta A_F = 0 \quad (2)$$

here

$$\delta A_F = - \int_{n=1}^{S_2} \sigma_{ij} \delta \varepsilon_{ij} dV - \int_{e=1}^{S_1} \Gamma_e \delta \Delta e$$

$$\delta A_u = - \int_{n=1}^{S_a} \rho_u \frac{\partial^2 \bar{u}}{\partial t^2} \delta u dV - \sum_{k=1}^n m_k \frac{d^2 u}{dt^2} \delta \bar{u}_k - \sum_{k=1}^n I_k \frac{d^2 u}{dt^2} \delta \varphi_k$$

$$\delta A_I = - \int_{n=1}^{S_2} \rho_n \bar{f} \delta \bar{u} dV + \int_{n=1}^{S_1} \bar{f} \delta \bar{u} dV + \sum_{n=1}^N F \delta \bar{u}_n + \sum_{k=1}^N m_k \delta \bar{\varphi}_k$$

$\delta \varepsilon_{ij}$ ,  $\delta e$  - strain variations of distributed and linear lumped elements;

**Fig. 1. Design scheme**

$\rho_n$  - material density of the  $n$ th lumped element;  $m_k$  - weight  $k$  – go hard body;  $u, u_k, \delta u_1, \delta u_k$  - vectors of displacements of points of distributed elements and centers of mass of rigid bodies and their variations;  $\bar{f}, \rho$  -Densities of mass and surface forces applied to distributed elements;  $V_n, E_n$  - volume and surface of the  $n$ th distributed element;  $I_n$  - tensor of the central moments of inertia of the  $n$ th solid;  $F_m, M_k$  - the main vector and the main moment

of forces applied to the solid body. As an example, we consider a body mounted on deformable supports (Fig. 1) or an object is considered a dissipative mechanical system. System (2) is written in matrix form with respect to the matrix – column  $\{X\} = \text{colon } (x_1, \dots, x_n)$

in the following way:

$$[M]\{\ddot{X}\} + [C]\{\dot{X}\} + [K]\{X\} = \{f\} \quad (3)$$

where the inertia matrix [M], damping matrix [C] and stiffness matrix [K] are n-order symmetric matrices. The perturbation is described by a column matrix.  $\{f\}$ . The physical meaning of matrix coefficients is as follows:  $M_{jk}$  - component of the momentum along  $j$  at unit speed no  $k$ ,  $C_{jk}$  - damping force over  $j$  at a unit speed in  $k$ ,  $K_{jk}$  - elastic force  $j$ , due to a single movement along  $k$ .

If the excitation matrix  $\{f\} = 0$ , then equation (3) describes the free oscillations of the system, and if  $\{f\} \neq 0$  - then forced. The solution of equation (3) can be hiccuped as

$$\{X(t)\} = \{W\}e^{\lambda t} \quad (4)$$

Where  $\lambda$  – complex number,  $W$ - complex numerical matrix - column. The numbers  $\lambda$  called characteristic indicators, and numbers  $i\lambda$  (or  $-i\lambda$ ) – complex frequencies. Characteristic indicators must be the roots of the characteristic equation

$$\det[[M]\lambda^2 + [C]\lambda + [K]] = 0 \quad (5)$$

A system with n degrees of freedom has 2n characteristic indicators  $\lambda_1, \dots, \lambda_{2n}$ .

If all characteristic exponents are the simple roots of equation (5), then the general solution of equation (5) will be equal to the sum of 2n partial solutions of the form

$$\{X(t)\} = \sum_{k=1}^{2n} C_k \{W_k\} e^{-i\lambda_k t} \quad (6)$$

Here  $C_k$  – arbitrary complex constants, and  $W_k$  – numeric matrices are columns.

We will present characteristic indicators in the form

$$\begin{aligned} \lambda_k &= \omega_{Rk} - i\omega_{Ik} \\ \lambda_{n-k} &= \omega_{Rk} + i\omega_{Ik} \quad , \quad (k = 1, \dots, n) \end{aligned} \quad (7)$$

Where  $\omega_{Ik} > 0$  and  $\omega_{Rk} > 0$  – real numbers called damping coefficients and natural frequencies of the damped system, respectively. If  $\{W_k\}$  and  $\lambda_k$  satisfy equation (5), then complex conjugate  $\{W_k^c\}$  and  $\lambda_k^c$  also satisfy him. When there is no damping, all roots lie on the imaginary axis. When damping, the roots are near the imaginary axis. If the system is dissipative and has complete dissipation, then all characteristic exponents lie in the lower half-plane of the complex variable. All particular solutions are decaying functions and, therefore, the general solution is a decaying function of time. If the system has incomplete dissipation, then part of its indicators lie in the left half-plane, and part on the imaginary axis. Among particular solutions, there are periodic ones that correspond to non-damped degrees of freedom. If the system has negative dissipation, then among the characteristic indicators there may be those whose real parts are negative. The corresponding particular and general solutions will be functions unlimitedly increasing in time.

### Calculation results

As an illustration, a block diagram of a dynamic vibration damper corresponding to the scheme of the first group is presented in Fig. 1. Taking into account the coupling in the motion

of dynamic dampers leads to variations in the parameters of the dynamic suppression regime and other related characteristics. However, the general dynamic properties of the system remain essentially unchanged when the number of resonances and the number of dynamic suppression modes are considered [12,13].

As a specific application of the proposed approach, the capabilities of a dynamic vibration damper obtained using a generalized methodology for constructing mathematical models of mechanical systems are analyzed (Fig. 1). The amplitude–frequency characteristic of a system equipped with a dynamic lever-type damper is illustrated in Fig. 2. The differential equation describing the motion of the system (1), as well as the corresponding transfer function, can be written in the following form.

$$W(p) = \frac{\bar{y}}{z} = \frac{mi(i+1)p^2 + k\bar{\Gamma}}{(M + mi^2)p^2 + k\bar{\Gamma}}$$

Fig. 4. The amplitude-frequency characteristics of the system at different mass ratios: curve *a* meets the condition  $M > mi$ ; curve *b* according  $M = mi$ ; the curve corresponds to  $M < mi$ .

The main material of the printed circuit board is fiberglass or getinax. Recently, in the electronic industry all fiberglass is becoming very popular, therefore, we will set initial conditions corresponding to this material:

- Young's modulus (E), equal to 105 kgf / m<sup>2</sup>,
- density (ρ) equal to 1400 kg / m<sup>3</sup>,
- frequency (f) equal to 200 Hz,
- **the amplitude of the external vibration(A<sub>0</sub>)**

Fig. 5. A change in the amplitude of movement, depending on the frequency, for different M. (1. M\* = 0.01, 2.- M\* = 0.1, 3. M\* = 1.0)

**The obtained system of differential equations describing the complex mechanical system is solved by applying the Laplace integral transform method [14].**

The results of the calculations are presented in Figures 4 and 5. Figure 5 illustrates three cases corresponding to different values of the dimensionless mass parameter:

1.  $M^* = 0.01$ ,
2.  $M^* = 0.1$ ,
3.  $M^* = 1.0$ .

Here the parameter is defined as  $M^* = (m_1 + m_2)/M$ . The results demonstrate that as the mass of the dynamic vibration damper increases, the oscillatory behavior of the system becomes more optimized and the vibration amplitude decreases.

To analyze the specific characteristics of dynamic damping in systems with mechanical linkages, theoretical investigations were conducted and subsequently compared with experimental observations [15]. The experimental study was performed on a prototype system equipped with vibration protection devices that include a mechanism for motion transformation [16].

Based on the conducted research, several conclusions can be formulated. A generalized variational mathematical formulation of the dynamic problem for dissipatively homogeneous and heterogeneous mechanical systems has been developed, particularly in relation to vibration

protection systems. In this formulation, the relationship between stresses and strains is taken into account using the Boltzmann–Volterra integral representation [17].

### **Conclusions**

A method for developing mathematical models of vibration protection systems based on the application of dynamic dampers with multiple degrees of freedom has been proposed. The dynamic characteristics of vibration dampers with different structural and technical configurations have been investigated.

A numerical–analytical analysis of oscillatory processes in nonlinear mechanical systems composed of spatial rigid bodies interconnected without massive viscoelastic elements has been carried out. In addition, the oscillatory behavior of nonlinear mechanical systems formed by a set of rectangular plates with concentrated masses and connected without massive viscoelastic components and supporting struts has been examined using numerical–analytical approaches.

Furthermore, a similar numerical–analytical investigation has been performed for nonlinear mechanical systems consisting of packages of cylindrical shells with concentrated masses, also interconnected without massive viscoelastic elements and structural struts. These studies provide additional insight into the dynamic response and vibration suppression capabilities of such mechanical configurations.

### **Literature**

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