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METHOD FOR SOLVING EXACT DIFFERENTIAL EQUATIONS

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Annotation

Exact differential equations constitute an important class of first-order differential equations widely used in mathematical analysis, physics, engineering, and applied sciences. These equations possess a special structure that allows them to be solved through the identification of a potential function whose total differential coincides with the given equation. The present study examines the theoretical foundations of exact differential equations and develops a systematic method for constructing their general solutions. Particular attention is given to the mathematical conditions under which a differential equation is exact and to the algorithm used for determining the potential function. The work analyzes the structural properties of exact equations and demonstrates the solution procedure through a detailed example. The results show that the method of exact equations provides an effective analytical tool for solving certain classes of nonlinear differential equations and contributes to a deeper understanding of differential relationships between variables.

Keywords

exact differential equation, total differential, potential function, integrating condition, differential equations, mathematical modeling.

Introduction

The theory of ordinary differential equations represents one of the fundamental areas of mathematical analysis concerned with relationships involving unknown functions and their derivatives. Differential equations arise naturally in the mathematical description of physical processes, engineering systems, economic models, and biological phenomena. Many scientific problems require determining a function whose derivatives satisfy a certain relationship.

Among first-order differential equations, exact differential equations occupy a special place because of their structural properties and their connection with the concept of total differentials. These equations allow the construction of a potential function whose differential coincides with the given equation. As a result, the problem of solving a differential equation can be reduced to determining an implicit function.

Exact differential equations appear in many branches of applied mathematics, particularly in thermodynamics, mechanics, and mathematical physics. Their solution method demonstrates important ideas such as the relationship between differential forms and potential functions.

The purpose of this study is to analyze the theoretical foundations of exact differential equations and to present a systematic algorithm for obtaining their general solution. The research also illustrates the solution process through a practical mathematical example.

The research aims to provide a comprehensive mathematical description of the solution process while maintaining clarity and theoretical rigor. The results contribute to a deeper understanding of nonlinear transformation methods and provide a useful framework for solving practical problems involving nonlinear differential equations.

Mathematical formulation of the Bernoulli differential equation

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Consider the first-order differential equation written in the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

where M(x,y) and N(x,y) are functions with continuous partial derivatives in a given region.

The equation is called an **exact differential equation** if there exists a function F(x,y) such that

$$dF = M(x,y)\,dx + N(x,y)\,dy$$

In this case

$$\frac{\partial F}{\partial x} = M(x,y), \qquad \frac{\partial F}{\partial y} = N(x,y)$$

For such a function to exist, the following **exactness condition** must hold:

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

If this condition is satisfied, the differential equation is exact.

Algorithm for solving exact differential equations

The procedure for solving an exact differential equation consists of the following steps.

Write the equation in the form

$$M(x,y)\,dx + N(x,y)\,dy = 0$$

Check the exactness condition

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

Find the potential function F(x,y) by integrating M(x,y) with respect to x:

$$F(x,y) = \int M(x,y)\,dx + g(y)$$

where g(y) is an unknown function.

Differentiate the obtained expression with respect to yyy and compare it with N(x,y) to determine g(y).

Substitute g(y) into F(x,y). The general solution is then obtained as

$$F(x,y) = C$$

where C is an arbitrary constant.

Example: Solve the differential equation

$$(2xy + y^2)\,dx + (x^2 + 2xy)\,dy = 0$$

Step 1. Identify the functions

$$M(x,y) = 2xy + y^2$$

$$N(x,y) = x^2 + 2xy$$

Step 2. Check the exactness condition

$$\frac{\partial M}{\partial y} = 2x + 2y$$

$$\frac{\partial N}{\partial x} = 2x + 2y$$

Since

$$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$$

the equation is **exact**.

Step 3. Find the potential function

Integrate M(x,y) with respect to x:

$$F(x,y) = \int (2xy + y^2)\, dx$$

$$F(x,y) = x^2y + xy^2 + g(y)$$

Step 4. Determine g(y)

Differentiate F(x,y) with respect to y:

$$\frac{\partial F}{\partial y} = x^2 + 2xy + g'(y)$$

Since

$$N(x,y) = x^2 + 2xy$$

we obtain

$$g'(y) = 0$$

$$g(y) = C$$

Step 5. General solution

Thus

$$F(x,y) = x^2y + xy^2$$

The general solution of the differential equation is

$$x^2y + xy^2 = C$$

Exact differential equations provide an important analytical framework for solving first-order differential equations that arise in various scientific applications. The existence of a potential function allows the equation to be interpreted as the total differential of a function of two variables. This interpretation simplifies the solution process and reveals deeper connections between differential equations and multivariable calculus.

The exactness condition plays a crucial role in identifying whether a differential equation can be solved using this method. When the condition is satisfied, the solution procedure becomes systematic and straightforward. In cases where the condition is not satisfied, integrating factors may sometimes be used to transform the equation into an exact form.

The method of exact equations therefore illustrates a broader mathematical principle: many differential problems can be solved by identifying underlying structural properties that simplify the analysis.

Conclusion

Exact differential equations represent a significant class of first-order differential equations characterized by the existence of a potential function whose total differential corresponds to the given equation. This property allows the construction of the general solution through integration and the identification of an implicit function.

The study has presented the theoretical foundation of exact differential equations and has developed a clear algorithm for solving them. A detailed example has demonstrated the practical application of the method and illustrated the step-by-step solution process.

The results confirm that the method of exact differential equations is an effective tool in the theory of ordinary differential equations and plays an important role in mathematical modeling and applied analysis.

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