

**INVESTIGATION OF NATURAL OSCILLATIONS IN LONGITUDINALLY
REINFORCED CYLINDRICAL SHELLS CONTAINING AN ELASTIC FILLER**

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Abstract

In this study, the vibration behavior of a thin, long cylindrical shell reinforced in the longitudinal direction and containing an elastic filler was analyzed using the variational approach. The investigation considered axial reinforcement as well as the effect of friction arising at the contact interface between the shell and the filler. The dependence of the natural vibration period on the formation of circumferential wave modes was determined while accounting for the frictional interaction between the shell structure and the elastic core. The obtained results indicate that the vibration characteristics are practically independent of the material parameters, since the relationship between the vibration frequency and Poisson's ratio does not depend on the value of the elastic modulus.

Keywords

connection, joint, modulus of elasticity, deformation, total energy, coefficient of friction.

Introduction: Cylindrical shells containing different types of fillers are extensively applied in many areas of mechanical engineering. Their widespread use requires a more comprehensive analysis of both material properties and structural characteristics in order to ensure rational design and reliable strength assessment. For an accurate evaluation of the load-carrying capacity of such structures, it is important to consider the external forces generated by the filler material. One of the significant effects arises from the contact interaction between the shell and the elastic medium.

The forces exerted by the filler can be interpreted as surface forces acting along the interface between the cylindrical shell and the elastic core. This interaction is rather complex and depends on a number of parameters, including the mechanical properties of the filler material and the surface characteristics of the shell. Among these factors, frictional forces caused by the relative interaction between the shell and the filler play a particularly important role.

The mathematical treatment of such problems is associated with considerable difficulties, especially when dynamic effects must also be taken into account. These effects often appear in engineering problems related to seismic resistance, vibration analysis, and other technical applications. Therefore, the development of reliable approximate computational approaches for such problems becomes highly relevant. One of the effective tools for this purpose is the variational method employed in this study. This method makes it possible to formulate consistent approximate principles for the analysis of thin-walled structural elements, including shells and rods.

It should be noted that most of the solutions available in the scientific literature deal primarily with reinforced cylindrical shells without internal fillers [1–3]. The vibration behavior of smooth cylindrical shells containing a filler has been investigated in detail in studies [4–7]. Meanwhile, the vibration characteristics of cylindrical shells reinforced with longitudinal ribs and interacting with an elastic medium have been considered in works [8–10].

Methods. This paper investigates the free vibration behavior of cylindrical shells containing an elastic filler and reinforced with discretely arranged longitudinal stiffeners under

axial compressive loading. Particular attention is given to the frictional interaction occurring at the contact interface between the shell and the filler material. The influence of the surrounding medium parameters on the natural vibration frequencies of the system is analyzed.

To address this problem, an energy-based approach is employed. Within this framework, the potential energy of the cylindrical shell subjected to axial compressive forces can be expressed as follows [11–12]:

$$\begin{aligned} \mathcal{O} = & \frac{Eh}{2(1-\nu^2)} \int_0^{\xi_1} \int_0^{2\pi} \left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \theta} - w \right)^2 + 2(1-\nu) \frac{\partial u}{\partial \xi} \frac{\partial v}{\partial \theta} - w \left(\frac{\partial u}{\partial \xi} + \frac{\partial v}{\partial \theta} - w \right) \\ & - \frac{1}{4} \left(\frac{\partial u}{\partial \theta} + \frac{\partial v}{\partial \xi} \right)^2 d\xi d\theta + \frac{Eh}{24(1-\nu^2)R^2} \int_0^{\xi_1} \int_0^{2\pi} \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta} \right)^2 \\ & - 2(1-\nu) \frac{\partial^2 w}{\partial \xi^2} \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta} \left(\frac{\partial^2 w}{\partial \xi^2} + \frac{\partial^2 w}{\partial \theta^2} + \frac{\partial v}{\partial \theta} \right) d\xi d\theta + \\ & + \frac{E_c}{2R} \sum_{i=1}^k F_c \frac{\partial u}{\partial \xi} - \frac{h_c}{R} \frac{\partial^2 w}{\partial \xi^2} + \frac{I_{yc}}{R^2} \frac{\partial^2 w}{\partial \xi^2} + \frac{G_c}{E_c} I_{kp.c} \frac{G_c}{E_c} I_{kp.c} \frac{\partial^2 w}{\partial \xi \partial \theta} + \frac{\partial v}{\partial \xi} \Big|_{\theta=\theta_1} d\xi - \\ & - \frac{\sigma_x h}{2} \int_0^{\xi_1} \int_0^{2\pi} \frac{\partial w}{\partial \xi} d\xi d\theta - \frac{\sigma_x F_c}{2R} \sum_{i=1}^k \frac{\partial w}{\partial \xi} \Big|_{\theta=\theta_1} d\xi \end{aligned} \quad (1)$$

Here $\xi_1 = \frac{L}{R}$, $\xi = \frac{x}{R}$, $\theta = \frac{y}{R}$; x, y, z - coordinates, E_c, G_c - elasticity and shear moduli of the longitudinal ribs material, k – number of longitudinal ribs, σ_x - axial compressive stresses, u, v, w - components of the shell displacement vector, h and R – the thickness and radius of the shell, respectively, E, ν - Young's modulus and Poisson's ratio of the shell material, $F_c, I_{yc}, I_{kp.c}$ - respectively, the areas and moments of inertia of the cross-section of the longitudinal rod relative to the axis OX and OZ , and also the moment of inertia during torsion.

The kinetic energy of the shell is:

$$\begin{aligned} K = & \frac{Eh}{2(1-\nu^2)} \int_0^{\xi_1} \int_0^{2\pi} \left(\frac{\partial u}{\partial t_1} + \frac{\partial v}{\partial t_1} + \frac{\partial w}{\partial t_1} \right)^2 d\xi d\theta \\ & + \frac{\overline{\rho_c} E_c F_c}{2R(1-\nu^2)} \sum_{i=1}^{k_1} \left(\frac{\partial u}{\partial t_1} + \frac{\partial w}{\partial t_1} \right)^2 \Big|_{\theta=\theta_1} d\xi \end{aligned} \quad (2)$$

Here $\overline{\rho_c} = \frac{\rho_c}{\rho_0}$, where ρ_0, ρ_c - the densities of the shell and longitudinal rod materials, respectively, $\theta_i = \frac{2\pi}{k_1} i$.

The interaction of the filler with the shell is represented as a surface load applied to the shell, which performs work on the displacements of the contact surface when transferring the system from a deformed state to the initial undeformed state.

$$A_0 = - \int_0^{\xi_1} \int_0^{2\pi} (q_x u + q_\theta v + q_z w) d\xi d\theta + \int_0^{\xi_1} \int_0^{2\pi} f q_z (u + v) d\xi d\theta \quad (3)$$

were q_x, q_θ, q_z - pressure from the filler on the shell, f - coefficient of friction.

The total energy of the system is:

$$\Pi = \mathcal{D} + K + A_0 \quad (4)$$

The equation of motion of the medium in vector form has the form

$$[2,3]: a_e^2 \text{grad div } \vec{S} - a_t^2 \text{rot rot } \vec{S} + \omega^2 \vec{S} = 0, \quad 0 \leq x \leq L, \quad 0 \leq r \leq R \quad (5)$$

Were $a_t^2 = (\lambda + 2\mu) / \rho$, $a_e^2 = \mu / \rho$, a_t, a_e - the propagation speeds of longitudinal and transverse waves in the filler, respectively; $S = S(S_x, S_\theta, S_z)$ - displacement vector; λ, μ - Lamé coefficients. Contact conditions are added to the systems of equations of motion of the medium (5). It is assumed that the contact between the shell and the filler is rigid, i.e. when $r = R$:

$$u = S_x; \quad v = S_\theta; \quad w = S_z \quad (6)$$

$$q_x = -\sigma_{rx}, \quad q_y = -\sigma_{r\theta}, \quad q_z = -\sigma_{rr}, \quad w = S_r \quad (7)$$

Components $\sigma_{rx}, \sigma_{r\theta}, \sigma_{rr}$ - stress tensors are defined as follows [13-15]:

$$\sigma_{rx} = \mu_s \left(\frac{\partial S_x}{\partial r} + \frac{\partial S_r}{\partial x} \right); \quad \sigma_{r\theta} = \mu_s \left(r \frac{\partial}{\partial r} \left(\frac{S_r}{r} \right) + \frac{1}{r} \frac{\partial S_r}{\partial \theta} \right), \quad (8)$$

$$\sigma_{rr} = \lambda_s \left(\frac{\partial S_r}{\partial x} + r \frac{\partial}{\partial r} \left(\frac{S_r}{r} \right) + \frac{1}{r} \frac{\partial S_\theta}{\partial \theta} \right) + 2\mu_s \frac{\partial S_r}{\partial r}$$

λ_s, μ_s - Lamé coefficients for the environment.

Supplementing the equations of motion of the filler (5) with contact conditions (6) and (7), we arrive at a contact problem of vibrations of a cylindrical shell reinforced with cross-rib systems filled with a medium. In other words, the problem of vibrations of a cylindrical shell with a filler reinforced with cross-rib systems under axial compression is reduced to the joint integration of the equations of shell theory and the equations of motion of the filler when the specified conditions are met on the surface of their contact.

Further, we will consider shells whose edges are hinged. We seek the components of the displacement vector of such shells in the form:

$$\begin{aligned} u &= A \cos kx \cos n\varphi \exp(i\omega_1 t_1), \\ \vartheta &= B \sin kx \sin n\varphi \exp(i\omega_1 t_1), \\ w &= C \sin kx \cos n\varphi \exp(i\omega_1 t_1) \end{aligned} \quad (9)$$

Where, A, B, C - unknown constants; $k = \frac{m\pi}{L}$ ($m = 1, 2, \dots$), m, n - wave numbers in the longitudinal and circumferential directions, respectively, L - length of the shell,

$$\omega_1 = \frac{\omega}{\omega_0}, \quad t_1 = \omega_0 t, \quad \omega_0 = \sqrt{\frac{E}{(1-\nu^2)\rho_0 R^2}}, \quad \omega_1 = \sqrt{\frac{(1-\nu^2)\rho_0 R^2 \omega^2}{E}}$$

For equal weights of the reinforced shell and the shell without reinforcement, their natural frequencies are denoted by ω and ω_0 .

The solutions of system (5) have the form [4, 15]:

a) with small inertial effects from the filler on the process of system oscillations:

$$\begin{aligned}
 S_x &= -kr \frac{\partial I_n(kr)}{\partial r} - 4(1-\nu_s)kI_n(kr) A_s + kI_n(kr)B_s \cos n\varphi \cos kx \exp(i\omega_1 t_1) \\
 S_\varphi &= -\frac{n}{r}I_n(kr)B_s - \frac{\partial I_n(kr)}{\partial r} \gamma_1 r C_s \sin \varphi \cos kx \exp(i\omega_1 t_1) \\
 S_r &= -k^3 r I_n(kr)A_s + \frac{\partial I_n(kr)}{\partial r} B_s + \frac{n}{r}I_n(kr)C_s \cos n\varphi \sin kx \exp(i\omega_1 t_1)
 \end{aligned} \tag{10}$$

b) the inertial effects of the filler on the process of system oscillations are significant:

$$\begin{aligned}
 S_x &= A_s k I_n(\gamma_e r) - \frac{C_s \gamma_i^2}{\partial r} I_n(\gamma_1 r) \cos n\varphi \cos kx \exp(i\omega_1 t_1) \\
 S_\varphi &= -\frac{A_s n}{r} I_n(\gamma_e r) - \frac{C_s n k}{r \mu} I_n(\gamma_1 r) - \frac{B_s}{n} \frac{\partial I_n(\gamma_1 r)}{\partial r} \sin n\varphi \sin kx \exp(i\omega_1 t_1) \\
 S_r &= A_s \frac{\partial I_n(\gamma_e r)}{\partial r} - \frac{C_s k}{\mu_1} \frac{\partial I_n(\gamma_1 r)}{\partial r} + \frac{B_s}{r} I_n(\gamma_1 r) \cos n\varphi \sin kx \exp(i\omega_1 t_1)
 \end{aligned} \tag{11}$$

Here I_n - modified Bessel function of the n th order of the first kind, A_s, B_s, C_s - permanent.

Using contact conditions (6), displacements of shells (9), solution of the equation of motion of the medium (10) and (11), we express the constants A_s, B_s, C_s through A, B, C . As a result, for q_x, q_θ, q_r we find:

$$\begin{aligned}
 q_x &= (\tilde{C}_{x1}A + \tilde{C}_{x2}B + \tilde{C}_{x3}C) \cos n\varphi \cos kx \exp(i\omega_1 t_1) \\
 q_\theta &= (\tilde{C}_{\theta1}A + \tilde{C}_{\theta2}B + \tilde{C}_{\theta3}C) \sin n\varphi \sin kx \exp(i\omega_1 t_1) \\
 q_r &= (\tilde{C}_{r1}A + \tilde{C}_{r2}B + \tilde{C}_{r3}C) \cos n\varphi \sin kx \exp(i\omega_1 t_1)
 \end{aligned} \tag{12}$$

После подстановки (12) в (3) и интегрирования по ξ и θ получаем для работы распределенных нагрузок со стороны заполнителя, приложенных к оболочке:

$$\begin{aligned}
 A &= -R^2 \pi [S_2 \tilde{C}_{x1} A^2 + (S_2 \tilde{C}_{x2} + S_1 \tilde{C}_{\theta1}) AB + (S_2 \tilde{C}_{x3} + S_1 \tilde{C}_{r1}) AC + \\
 &+ S_1 (\tilde{C}_{\theta3} + \tilde{C}_{r2}) BC + S_1 \tilde{C}_{\theta2} B^2 + S_1 \tilde{C}_{r3} C^2
 \end{aligned} \tag{13}$$

Here \tilde{C}_{ra} - constant, $S_1 = \frac{1}{2} - \frac{\sin 2k\xi_1}{4k}$.

Using (1), (2), (13) for the total energy of the system we obtain a second-order polynomial with respect to the constant parameters A,B,C:

$$\begin{aligned}
 \Pi &= (\tilde{\varphi}_{11} - S_2 \tilde{C}_{x1} - \psi_{11} \omega_1^2) A^2 + (\tilde{\varphi}_{22} - S_1 \tilde{C}_{\theta2} - \psi_{22} \omega_1^2) B^2 + (\tilde{\varphi}_{33} - S_1 \tilde{C}_{r3} - \psi_{33} \omega_1^2 + I_1 \sigma_x) C^2 + \\
 &+ (\tilde{\varphi}_{44} - S_2 \tilde{C}_{x2} + S_1 \tilde{C}_{\theta1}) AB + (\tilde{\varphi}_{55} - S_2 \tilde{C}_{x3} + S_1 \tilde{C}_{r1}) AC + S_1 (\tilde{\varphi}_{66} + \tilde{C}_{\theta3} + \tilde{C}_{r2}) BC
 \end{aligned}$$

Note that the quantities $\tilde{\varphi}_{ii} (i=1,2,\dots,6)$, $\psi_{ii} (i=1,2,\dots,6)$, $I_i (i=1,2)$ have a bulky appearance, so we do not include them here.

The conditions of the extremum P for the parameters A, B, C reduce the solution of the problem of vibrations of a shell reinforced by longitudinal systems of ribs filled with a medium and subjected to longitudinal compression, taking into account friction in contact, to homogeneous systems of linear algebraic equations of the third order, non-trivial solutions of

which are possible only if the determinant of this system is equal to zero. Equating the determinants of the indicated systems to zero, we obtain the following frequency equation:

$$\begin{aligned} 2(\check{\varphi}_{11} - S_2\check{C}_{x1} - \psi_{11}\omega_1^2)A + (\check{\varphi}_{44} + S_2\check{C}_{x2} + S_1\check{C}_{\theta1})B + (\check{\varphi}_{55} - S_2\check{C}_{x3} + S_1\check{C}_{r1})C &= 0 \\ (\check{\varphi}_{44} + S_2\check{C}_{x2} + S_1\check{C}_{\theta1})A + 2(\check{\varphi}_{22} - S_1\check{C}_{\theta2} - \psi_{22}\omega_1^2)B + (\check{\varphi}_{66} + \check{C}_{\theta3} + \check{C}_{r2})C &= 0 \\ (\check{\varphi}_{55} + S_2\check{C}_{x3} + S_1\check{C}_{r1})A + (\check{\varphi}_{66} + \check{C}_{\theta3} + \check{C}_{r2})B + 2(\check{\varphi}_{33} - S_1\check{C}_{r3} - \psi_{33}\omega_1^2 + I_1\sigma_x)C &= 0 \end{aligned} \quad (14)$$

It is easy to see that in case a) the system of equations (14) is reduced to a cubic equation with respect to ω ω_1^2 , otherwise it is transcendental. Since in what follows we will be interested only in low frequencies of bending vibrations, this equation in case a) can be simplified by discarding the terms with ω_1^4 and ω_1^6 . As a result we get ($\omega_1^2 = \lambda_a$):

$$\begin{aligned} \lambda_a &= \frac{f_3^2 f_4 + f_1 f_5^2 + f_2^2 f_6}{2f_5^2 \psi_{11} + f_2^2 \psi_{33} - 4f_1 f_4 \psi_{33} - 0,5f_6(f_1 \psi_{22} + f_4 \psi_{11})} \\ f_1 &= \check{\varphi}_{11} - S_2\check{C}_{x1}; \quad f_2 = \check{\varphi}_{44} + S_2\check{C}_{x2} + S_1\check{C}_{\theta1}; \quad f_3 = \check{\varphi}_{55} + S_2\check{C}_{x1} + S_1\check{C}_{r1}; \\ f_5 &= \check{\varphi}_{66} + \check{C}_{\theta3} + \check{C}_{r2}; \quad f_6 = \check{\varphi}_{33} - S_1\check{C}_{r3} + I_1\sigma_x \end{aligned} \quad (15)$$

It is defined in a similar way λ_b for the occasion b).

Results and analysis. Let us present the results of the study of the influence of the number of ribs and the rigidity of the fillers on the critical stress of axial compression. The calculations were performed for the shell, medium and ribs with the following parameters:

$$E = E_c = E_h = 6,67 \cdot 10^9 \text{ H / m}^2; \quad \nu = 0,3; \quad x = 1; \quad n = 8; \quad h_h = 1,39\text{mm}; \quad R = 160\text{mm};$$

$$L_1 = 800\text{mm}; \quad \frac{F_c}{2\pi R h} = 0,1591 \cdot 10^{-1}; \quad \frac{I_{yc}}{2\pi R^3 h} = 0,8289; \quad h = 0,45\text{mm};$$

$$F_x = 5,75\text{mm}^2; \quad I_{sh} = 19,9\text{mm}^4; \quad |h_c| = 0,1375 \cdot 10^{-1} R; \quad \frac{I_{kpc}}{2\pi R^3 h} = 0,5305 \cdot 10^{-6};$$

$$I_{kph} = 0,48\text{mm}^4; \quad f = 0,25$$

The calculation results are presented in Fig. 1. The dependence of the axial compression stress is shown here. From Fig. 1 it is evident that with increasing stress the frequency of the system decreases. In addition, taking into account friction leads to a decrease in the value of the natural frequency of the structure under study. As noted, the method for determining the optimal reinforcement parameters is based on a comparison of the minimum vibration frequencies of a ribbed and smooth cylindrical shell, reinforced by longitudinal rib systems filled with a medium.

The following parameters are considered as variable: relative thickness of the shell $h^* = h / R$, distances between longitudinal and transverse ribs, related to the thickness of the shell ratio of the weight of all ribs to the weight of the shell φ_1 and the ratio of the weight of the longitudinal ribs to the weight of the transverse ribs φ_2 . It is assumed that the radius and length of the shell, as well as the characteristics of the shape of the sections of the longitudinal and transverse ribs are predetermined. Note that for rectangular sections it is necessary to specify the relations ψ_1 and ψ_2 heights of longitudinal and annular ribs to their thicknesses, respectively. The dimensionless characteristics of the ribs included in (1), (2) are expressed through the specified parameters:

$$\begin{aligned} \bar{\gamma}_c^{(1)} &= \frac{\varphi_1 \varphi_2}{1 + \varphi_2}, \quad \bar{\gamma}_s^{(2)} = \frac{\varphi_1}{1 + \varphi_2}, \quad \frac{h_c}{R} = -\frac{h^*}{2} (1 + \sqrt{a_1 \varphi_1 \bar{\gamma}_c^{(1)}}), \\ \mu_{s2} &= \frac{1-\nu}{6} \frac{a_2}{\psi_2} (h^*)^2 (\bar{\gamma}_s^{(2)})^2; \quad \frac{h_c}{R} = -\frac{h^*}{2} \left(1 + \frac{1}{k_1} \sqrt{a_1 \varphi_1 \bar{\gamma}_c^{(1)}} \right), \\ \eta_{s1}^{(2)} &= \bar{\gamma}_{s1}^{(2)} \bar{\gamma}_s^{(2)} \frac{a_2 \psi_2 (h^*)^2}{12}, \quad \eta_{s1}^{(2)} = \bar{\gamma}_{s1}^{(2)} \bar{\gamma}_s^{(2)} \frac{a_2 \psi_2 (h^*)^2}{12}, \\ \eta_c^{(1)} &= \bar{\gamma}_c^{(1)} \frac{a_1}{12} \psi_1 \bar{\gamma}_c^{(1)} (h^*)^2 + \frac{h_c}{R}^2, \quad \mu_{s1} = \frac{1-\nu}{6} (h^*)^2 (\bar{\gamma}_c^{(1)})^2 \frac{a_1}{\psi_1} \end{aligned}$$

With this formulation, the result of the study is practically independent of the characteristics of the shell material, since (ω_{\min}^2) , as is known, weakly depend on Poisson's ratio ν , and their attitude μ do not depend on the modulus of elasticity E . It should be noted that in order to improve the bearing capacity of the shell, it is necessary to find such a combination of parameters h^* , a_1 , a_2 , φ_1 and φ_2 , under which μ takes on the greatest value.

As an example to illustrate the changes μ Depending on the relative weights of the ribs, the results of calculations of cylindrical shells filled with a medium reinforced by longitudinally supported rib systems are presented.

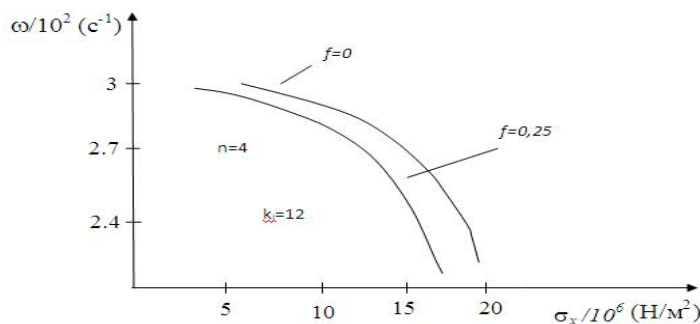


Fig. 1. System frequency dependencies $\Omega = \Omega_1 \Omega_0$ from compressive stresses

4. Conclusions

The results of the study are practically independent of the characteristics of the shell material, since (ω_{\min}^2) weakly dependent on Poisson's ratio ν , and their attitude μ do not depend on the modulus of elasticity E . It has been established that in order to improve the bearing capacity of the shell, it is necessary to find such a combination of parameters h^* , a_1 , a_2 , φ_1 и φ_2 , under which μ takes on the greatest value.

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