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CONSTRUCTING NONLOCAL FUNDAMENTAL SPLINES

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Abstract

In this work, we study the problem of constructing nonlocal fundamental splines on rectangular finite elements. In this case, we use the coefficients of the algebraic-trigonometric optimal interpolation formula constructed using the Sobolev method in the Hilbert space of differentiable functions. In addition, we will prove the theorem expressing the main property of these nonlocal fundamental splines.

Keywords

Interpolation, optimal interpolation formula, error functional, Hilbert space, approximation, fundamental spline, finite elements, Sobolev method.

Let us be given a domain $\Omega=[0,1]\quad[0,1]\qquad{}^{2}$ and a mesh

$$\Delta_{h_1,h_2}:=$$
$$=\{(x_i,y_j):x_i=ih_1,y_j=jh_2,i=0,1,...,m,j=0,1,...,n,h_1m=1,h_2n=1\}$$

in this domain. Through points x_i and y_j of mesh Δ_{h_1,h_2}, we draw lines parallel to the ordinate and abscissa axes, respectively. As a result, the domain Ω is divided into rectangular parts (i.e., finite elements) of the same shapes $[x_i,x_{i+1}]\quad[y_j,y_{j+1}]\,(i=0,1,...,m-1,j=0,1,...,n-1)$.

To construct fundamental splines of two variables on finite elements $[x_i,x_{i+1}]\quad[y_j,y_{j+1}]\,(i=0,1,...,m-1,j=0,1,...,n-1)$ in the domain Ω, we use the concept of the tensor product of functions of one variable.

Definition 1. Let f and g be functions of one variable. We define a two-variable function $f\quad g$ using the expression

$$f\quad g=f\quad g(x,y)=f(x)g(y). \tag{1}$$

The function $f\quad g$ of two variables is called the tensor product of the functions f and g of one variable. Where the symbol represents the tensor product [1].

In general, there are two methods for constructing fundamental splines of two variables. These are the construction method using the tensor product and the geometric method [1]. To simplify the calculations, we construct two-variable fundamental splines using the tensor product below. For this, we assign a function $\varphi_{i,j}(x,y)$, defined in the form

$$\varphi_{i,j}(x,y)=\varphi_i\quad\varphi_j(x,y)=\varphi_i(x)\varphi_j(y)\,(i=0,1,...,m;j=0,1,...,n)$$

and satisfying the relation

$$\varphi_{i,j}(x_p,y_q)=\begin{matrix}1,\ p=i\ \text{and}\ q=j,\\ 0,\ \text{otherwise}\end{matrix}$$

i.e.,

$$\varphi_{i,j}(x_p,y_q)=\delta_{i,p}\delta_{j,q}(p=0,1,...,m,q=0,1,...,n), \qquad (2)$$

to each node (x_i,y_j) . Where $\varphi_i \quad \varphi_j(x,y)$ is defined by (1), $\varphi_i(x)$ and $\varphi_j(y)$ are determined using optimal coefficients $C_i(x)\,(i=0,1,...,N)$ in the work [2], $\delta_{i,p}$ and $\delta_{j,q}$ are Kronecker symbols.

It should be noted that the functions $\varphi_{i,j}(x,y)(i=0,1,...,m,j=0,1,...,n)$ form a finite-dimensional linear space in the domain Ω, and therefore these functions are uniquely defined by their values at the nodes (x_i,y_j).

Through these linear transformations

$$x=-\frac{1}{a-b}s+\frac{a}{a-b}, y=-\frac{1}{a-b}t+\frac{a}{a-b},$$

it is possible to convert the domain Ω into any arbitrary domain $[a,b] \quad [a,b]$. Here a and b are some real numbers, and s and t are new variables.

Remark 1. Functions $\varphi_{i,j}(x,y)\,(i=0,1,...,m,j=0,1,...,n)$ defined in the domain Ω are fundamental splines of two variables.

The validity of Remark 1 follows from the definition of the fundamental spline [3] and from relation (2).

To geometrically represent the basis functions

$$\varphi_{i,j}(x,y)\,(i=0,1,...,m,j=0,1,...,n),$$

we present their graphs for the case $m=n=10$ and $\omega_1=\omega_2=1$ in Figure 1. Here, ω_1 and ω_2 are the parameters in the expression of the functions $\varphi_i(x)$ and $\varphi_j(y)$, respectively.

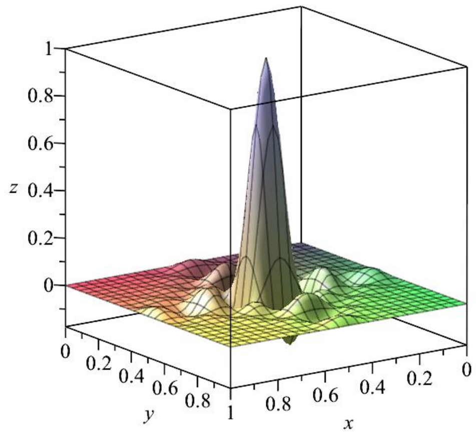

Figure 1. This figure shows the graph of the function $\varphi_{i,j}(x,y)\,(i=1,2,...,m-1,j=1,2,...,n-1)$.

Theorem 1. (Theorem on the main property) For nonlocal fundamental splines $\varphi_{i,j}(x,y)$ in the domain Ω, the following relations hold:

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,y)=1, \qquad (3)$$

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$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,y)\sin(\omega_1 x_i)\sin(\omega_2 y_j)=\sin(\omega_1 x)\sin(\omega_2 y), \quad (4)$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,y)\cos(\omega_1 x_i)\cos(\omega_2 y_j)=\cos(\omega_1 x)\cos(\omega_2 y), \quad (5)$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,y)\sin(\omega_1 x_i)\cos(\omega_2 y_j)=\sin(\omega_1 x)\cos(\omega_2 y), \quad (6)$$

$$\sum_{i=0}^{m}\sum_{j=0}^{n}\varphi_{i,j}(x,y)\cos(\omega_1 x_i)\sin(\omega_2 y_j)=\cos(\omega_1 x)\sin(\omega_2 y), \quad (7)$$

where ω_1 and ω_2 are elements of the set , $\{0\}$.

The proof of Theorem 1 follows from Definition 1 and from equalities (5) in the work [2].

From relations (3)-(7), it can be stated that the nonlocal fundamental splines $\varphi_{i,j}(x,y)\,(i=0,1,...,m, j=0,1,...,n)$ accurately reconstruct any linear combination of the functions 1, $\sin(\omega_1 x)\sin(\omega_2 y)$, $\cos(\omega_1 x)\cos(\omega_2 y)$, $\sin(\omega_1 x)\cos(\omega_2 y)$ and $\cos(\omega_1 x)\sin(\omega_2 y)$.

It should be noted that nonlocal fundamental splines $\varphi_{i,j}(x,y)\,(i=0,1,...,m, j=0,1,...,n)$ can be applied to finite element methods and signal processing.

Conclusion

In this article, a bounded region on a plane is divided into rectangular finite elements, and nonlocal fundamental splines are constructed on these finite elements. For this, the coefficients of the algebraic-trigonometric optimal interpolation formula constructed in the Hilbert space were used. Also, the theorem expressing the main property of these fundamental splines was proven.

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