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USING VIETA'S THEOREM IN SOLVING QUADRATIC EQUATIONS WITH PARAMETERS.

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Abstract: This article demonstrates solutions to various quadratic equations and quadratic equations with parameters using Vieta's theorem. The problems discussed in this article help to increase students' interest in mathematics, as well as to develop their skills of logical thinking and independent decision-making in problem situations.

Keywords: quadratic equation, parameter, theorem.

INTRODUCTION

In the current education system, it is important not only to develop students' level of knowledge, but also their ability to analyze acquired knowledge.

Studying quadratic equations with parameters teaches students not to complete test tasks using ready-made formulas, but to reason deeply and solve problems. Solving quadratic equations with parameters using Vieta's theorem improves students' skills in applying Vieta's theorem to solve examples.

Below we will explore examples of quadratic equations and quadratic equations with parameters along with their solutions.

Example 1. $x^2+ax+b=0$ the negative roots of the equation $x_1\ va\ x_2$ numbers $x_1{}^2+x_2{}^2=20$ satisfy the condition. $\dfrac{a}{\sqrt{b+10}}$ Find the value of.

A)-2 B) $-\sqrt{2}$ C) $\sqrt{2}$ D) 2

Solution: Using Vieta's theorem for the given $x^2+ax+b=0$ equation, we write the following system:

$$x_1+x_2=-a$$
$$x_1x_2=b$$

Now we write the given $x_1{}^2+x_2{}^2=20$ equality $(x_1+x_2)^2-2x_1x_2=20$ in the following form and substitute the values from the system above.

$a^2-2b=20$; $a^2=2b+20$

Therefore, the following holds

$$\frac{a}{\sqrt{b+10}}=\frac{\sqrt{2(b+10)}}{\sqrt{b+10}}=\sqrt{2}$$

Answer: $\frac{a}{\sqrt{b+10}}=\sqrt{2}$ i.e. C) $\sqrt{2}$

Example 2. $x^2-2ax+27=0$ the roots of the equation $x_1\ va\ x_2$ for numbers $\frac{1}{\sqrt[3]{x_2}}+\sqrt[3]{x_1}=4$ if the equality holds, a find the value of.

A)-28 B) -14 C) 14 D) 28

Solution: Let the roots of the given $\frac{1}{\sqrt[3]{x_2}}+\sqrt[3]{x_1}=4$ equality into the following form:

$1+\sqrt[3]{x_1x_2}=4\sqrt[3]{x_2}$. According to Vieta's theorem for $x^2-2ax+27=0$ we have $x_1+x_2=2a\ \ va\ \ x_1x_2=27$ hold. Now we substitute these values into $1+\sqrt[3]{x_1x_2}=4\sqrt[3]{x_2}$: $1+\sqrt[3]{27}=4\sqrt[3]{x_2}$ From this it follows that, $x_2=1$

$$x_1x_2=27; \qquad x_2=27\ .\ \ x_1+x_2=2a\ \ dan\ \ a=\frac{x_1+x_2}{2}=\frac{1+27}{2}=14$$

Answer: $a=14$ i.e. C)14

Example 3. Compose an equation whose roots are the $4x^3-7x^2-25x+21=0$ opposite numbers of the roots of the equation.

A) $4x^3+7x^2-25x-21=0$
B) $4x^3+7x^2+25x-21=0$
C) $4x^3+7x^2-25x+21=0$
D) $4x^3-7x^2+25x+21=0$

Solution: Let the roots of the given $4x^3-7x^2-25x+21=0$ equation be x_1, x_2, x_3 and let the roots of the required equation be x_1 , x_2 , x_3 . Then $x_1+x_1=0;\ \ x_2+x_2=0;\ \ x_3+x_3=0$ and

$$x_1+x_2+x_3=-\frac{7}{4}$$
$$x_1x_2+x_2x_3+x_1x_3=-\frac{25}{7}$$
$$x_1x_2x_3=-\frac{21}{7}$$

hold.

According to Vieta's theorem for the required cubic equation, the following hold

$$x_1\ +x_2\ +x_3\ =-x_1-x_2-x_3$$
$$x_1\ x_2\ +x_2\ x_3\ +x_1\ x_3\ =x_1x_2+x_2x_3+x_1x_3$$
$$x_1\ x_2\ x_3\ =-x_1x_2x_3$$

From this it follows that,
$$x_1\ +x_2\ +x_3\ =\frac{7}{4}$$
$$x_1\ x_2\ +x_2\ x_3\ +x_1\ x_3\ =-\frac{25}{7}$$
$$x_1\ x_2\ x_3\ =\frac{21}{7}$$

Therefore, the required equation is:

$$x^3+\frac{7}{4}x^2-\frac{25}{4}x-\frac{21}{4}=0 \text{ i.e. } 4x^3+7x^2+25x-21=0$$

Answer: A) $4x^3+7x^2+25x-21=0$

Example 4. $x^3-(a-2)x^2+(a+2)x-4=0$ the roots of the equation $x_1, x_2\ va\ x_3$ be. If $x_1x_2=2$ then a find the value of.

A)-8 B)-6 C) 6 D) 8

Solution: Let the roots of the given $x^3-(a-2)x^2+(a+2)x-4=0$ equation, according to Vieta's theorem, $x_1x_2x_3=4$. According to the condition $x_1x_2=2$. Therefore, the $x_3=2$ root of the equation was found. We substitute this value into the given equation $2^3-(a-2)\ 2^3+(a+2)\ 2-4=0$. From this it follows that, $a=8$

Answer: D) 8

Example 5. $x^2-3x+1=0$ the roots of the equation $x_1\ va\ x_2$ are the numbers, find $x_1^5+x_2^5$.

A) 120 B) 123 C) 126 D) 129

Solution: Let the roots of the given $x^2-3x+1=0$ equation, we apply Vieta's theorem:

$$x_1+x_2=3$$
$$x_1x_2=1$$

$$x_1^3+x_2^3=(x_1+x_2)\left((x_1+x_2)^2-3x_1x_2\right)=18$$

$$x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=7$$

We multiply these 2 equalities. Then we expand the brackets

$$\left(x_1^3+x_2^3\right)\left(x_1^2+x_2^2\right)=18\ \ 7$$

$$x_1^5+x_2^5+x_1^3x_2^2+x_1^2x_2^3=126$$

From this equality $x_1^5+x_2^5=126-x_1^3x_2^2+x_1^2x_2^3=126-(x_1x_2)^2(x_1+x_2)=126-1^2\ \ 3=126-3=123$

Answer: $x_1^5+x_2^5=123$ i.e. B) 123

Example 6. If c and d are $x^2-x-4=0$ roots of the equation, find the value of c^6+d^6 the expression.

A) 41 B) 121 C) 249 D) 297

Solution: Let the roots of the given $x^2-x-4=0$ according to Vieta's theorem $x_1+x_2=1$, $x_1x_2=-4$ hold. According to the condition $c=x_1;\ \ d=x_2$.

Therefore, $c^2+d^2=x_1^2+x_2^2=(x_1+x_2)^2-2x_1x_2=1-2\ \ (-4)=9$

$$c^6+d^6=x_1^6+x_2^6=\left(\left(x_1^2\right)^3+\left(x_2^2\right)^3\right)\left(x_1^4+x_1^4-x_1^2x_2^2\right)=$$

$$=9\ \ \left(\left(x_1^2+x_2^2\right)^2-3x_1^2x_2^2\right)=9\ \ \left(9^2-3\ \ (-4)^2\right)=297$$

Answer: $c^6+d^6=297$ i.e. D) 297

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Example 7. $ax\mathrm{I}-7x+2>0$ inequality holds for x any value of a find the smallest natural value of.

Solution:

For the quadratic inequality to be positive for all x:

1) $a>0$

2) $D<0$

$$D=49-8a<0$$
$$49<8a$$
$$a>\frac{49}{8}=6.125$$

$a > 6.125$ and a — natural number

The smallest natural number: $a=7$

Answer: D)

Example 8. $x\mathrm{I}+(a+1)x+4 \quad 0$ find all values of $a_1+a_2=?$

Solution:

For the quadratic inequality ≤ 0 to have a unique solution, the parabola x must be tangent to the x-axis at one point.

This occurs when $D=0$.

$$D=(a+1)\mathrm{I}-16=0$$
$$(a+1)\mathrm{I}=16$$
$$a+1=\pm 4$$

$a=3$ yoki $a=-5$

$$a_1+a_2=-2$$

Answer: A) -2

Example 9

a For what values of $x\mathrm{I}-(2a+3)x+2a\mathrm{I}+a-3=0$ do the roots of the equation $x_1<3<x_2$ satisfy the condition?

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A) $(0;\ 3)$ B) $(-1/2;\ 3)$ C) $-\ ;\ -\frac{1}{2}$ $(3;\ \ )$ D) $(-\ ;\ 0)$ $(3;\ \ )$

Solution:

$x_1 < 3 < x_2$ For to hold, the parabola $x = 3$ must be negative at the point:

$$f(3) < 0$$

$$f(x) = x\text{I} - (2a+3)x + 2a\text{I} + a - 3$$
$$f(3) = 9 - 3(2a+3) + 2a\text{I} + a - 3$$
$$= 9 - 6a - 9 + 2a\text{I} + a - 3$$
$$= 2a\text{I} - 5a - 3$$

$2a\text{I} - 5a - 3 < 0$ we solve the inequality:

$$D = 25 + 24 = 49$$

$$a = \frac{5 \pm 7}{4}$$

$$a_1 = \frac{5-7}{4} = -1/2$$

$$a_2 = \frac{5+7}{4} = 3$$

$2a\text{I} - 5a - 3 < 0$ $\quad a \quad -\frac{1}{2};\ 3$

Answer: B) $-\frac{1}{2};\ 3$

Example 10.

a For what value of $ax\text{I} + 6x + 4 \quad 0$ does the solution of the inequality consist of exactly one number?

A) $a = -\frac{9}{4}$ B) $a = -\frac{4}{9}$ C) $a = \frac{9}{4}$ D) $a = -\frac{4}{9}$

Solution:

$ax\text{I} + 6x + 4 \quad 0$ For the inequality to have exactly one solution,

the parabola x must be tangent to the x-axis at one point.

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Conditions:

1) $a > 0$ (parabola opens upward)

2) $D = 0$ (one root)

$$D = 36 - 16a = 0$$
$$36 = 16a$$
$$a = \frac{36}{16} = \frac{9}{4}$$

Verification: $a = \frac{9}{4} > 0$.

Answer: C) $a = \frac{9}{4}$

Exercises for independent solving.

1. $x^3 - ax^2 + bx - 120 = 0$ the equation has 3 natural solutions, one of which equals 10. a Find the smallest value of.

A)16 B) 17 C)18 D)23

2. The numbers 1; 4; 5 are $x^3 + ax^2 + bx + c = 0$ roots of the equation. $ax^2 + bx + c = 0$ Find the product of the roots of the equation.

A)-20 B) -2 C) 2 D)20

3. For prime numbers p; q $x^2 + px + q = 0$ the roots of the equation are known to be integers. How many values does the expression p+q have for p and q satisfying this condition?

A)3 B)2 C)1 D) infinitely many

4. $ax^2 - 3ax + b = 0 \; (a \quad 0)$ be the roots of the equation a va b How many quadratic equations satisfying these conditions exist?

A)1 B) 2 C) 3 D) does not exist

5. $ax^2 + 4x + a - 1 = 0$ the equation has 2 real x_1 va x_2 roots. Find such a value of parameter b so that $(x_1 - b)(x_2 - b)$ the value of the expression a does not depend on the value of the parameter.

A) $\frac{1}{2}$ B) $-\frac{1}{2}$ C) $\frac{1}{4}$ D) $-\frac{1}{4}$

6. a, b va c numbers $x^3-3x^2-x+6=0$ roots of the equation, find the value of $\frac{a}{bc}+\frac{b}{ca}+\frac{c}{ab}$ the expression.

A) $-\frac{11}{6}$ B) $-\frac{7}{6}$ C) $\frac{7}{6}$ D) $\frac{11}{6}$

7. $x^3+4x^2-7x-2=0$ Find the sum of the squares of the roots of the equation.

A)2 B) 30 C) 9 D)23

8. $x^2-4x+2=0$ the roots of the equation x_1 va x_2 be the roots, compose a quadratic equation whose roots are $\frac{x_1}{x_2}+1$ va $\frac{x_2}{x_1}+1$.

A) $x^2-6x+6=0$

B) $x^2-7x+7=0$

C) $x^2-8x+8=0$

D) $x^2-9x+9=0$

9. If $ax^3-2x^2-5x-6=0$ one of the roots of the equation equals -2, find the sum of all real roots.

A)-4 B) -2 C) 2 D)4

10. If 2 and 3 $x^3+mx^2+n=0$ are the roots of the equation, find its third root.

A)-1, 4 B) -1,2 C) 1,2 D) 1,4

CONCLUSION.

It is known that mastering examples and problems involving parameters is somewhat difficult for students. If students are taught more examples like those above and practice them repeatedly, they can acquire the knowledge, skills, and competencies as with other topics. The problems discussed in this article also lay the groundwork for students to study inequalities with parameters.

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