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NON-STANDARD PROBLEMS ON SYSTEMS OF EQUATIONS

Madrimova Zulayxo Sobirovna

Academic Lyceum of Urganch State Uexpressversity,

highest-category mathematics teacher

INTRODUCTION

This article presents a complete set of problems and solutions for the system of equations. At the same time, several examples are recommended for independent solutions by students.

The problems in this article can also be recommended for independent solutions by gifted students of general education schools.

Below we present the solutions to several problems.

Example 1. $\begin{cases} a+2b=4ab, \\ 4b+3c=18bc, \\ 6c-a=3ac \end{cases}$ for the roots of the system of equations $a+b-c$

find the andlue of the expression. $(abc \quad 0)$

A) $\frac{5}{6}$ B) 1 C) $\frac{7}{6}$ D) $\frac{11}{6}$

Solution. To solve the given system of equations, we divide the first, second and third equations of the system by ab, bc and ac respectively, and we obtain the following .

$$\begin{cases} a+2b=4ab, \\ 4b+3c=18bc, \\ 6c-a=3ac \end{cases} \qquad \begin{cases} \frac{1}{b}+\frac{2}{a}=4, \\ \frac{4}{c}+\frac{3}{b}=18, \\ \frac{6}{a}-\frac{1}{c}=3 \end{cases}$$

system of equations is formed. Bundan

$$\begin{cases} \frac{1}{b}+\frac{2}{a}=4, \\ \frac{4}{c}+\frac{3}{b}=18, \\ \frac{6}{a}-\frac{1}{c}=3 \end{cases} \qquad \begin{cases} \frac{6}{a}-\frac{4}{c}=-6, \\ \frac{6}{a}-\frac{1}{c}=3 \end{cases}$$

system of equations.

To eliminate the unknown a from the resulting system of equations, we subtract them and find c we find:

$$-\frac{3}{c}=-9 \qquad c=\frac{1}{3}.$$

Using the found c , we find a and b the unknowns:

$$6c-a=3ac \qquad 6\cdot\frac{1}{3}-a=3\cdot\frac{1}{3}a \qquad 2a=2 \qquad a=1.$$

$$4b+3c=18bc \qquad 4b+3\cdot\frac{1}{3}=18\cdot\frac{1}{3}b \qquad 2b=1 \qquad b=\frac{1}{2}.$$

Therefore, $a=1,\ b=\frac{1}{2},\ c=\frac{1}{3}$.

$$a+b-c=1+\frac{1}{2}-\frac{1}{3}=\frac{7}{6}.$$

Answer: $\frac{7}{6}$.

Example 2. $\begin{cases} x+y=3xy, \\ y+z=2yz, \\ z+x=4zx \end{cases}$ from the system of equations z find the andlue of

A) $\frac{1}{4}$ B) $\frac{2}{3}$ C) $\frac{3}{2}$ D) 1

Solution. To solve the given system of equations, we divide the first, second and third equations of the system by xy, yz and xz respectively, and we obtain the following

$$\begin{aligned} x+y&=3xy,\\ y+z&=2yz,\\ z+x&=4zx, \end{aligned} \qquad \begin{aligned} \frac{1}{y}+\frac{1}{x}&=3,\\ \frac{1}{z}+\frac{1}{y}&=2,\\ \frac{1}{x}+\frac{1}{z}&=4 \end{aligned}$$

system of equations is formed.

To eliminate the unknown y from the system of equations, we subtract the first and second equations term by term, and we obtain

$$\begin{aligned} \frac{1}{y}+\frac{1}{x}&=3,\\ \frac{1}{z}+\frac{1}{y}&=2,\\ \frac{1}{x}+\frac{1}{z}&=4 \end{aligned} \qquad \begin{aligned} \frac{1}{x}-\frac{1}{z}&=1,\\ \frac{1}{x}+\frac{1}{z}&=4 \end{aligned}$$

ikki noma’lumli the system of equationsga we obtain.

From the resulting system of equations z to find, we subtract them:

$$\begin{aligned} \frac{1}{x}-\frac{1}{z}&=1,\\ \frac{1}{x}+\frac{1}{z}&=4 \end{aligned} \qquad \frac{2}{z}=2 \qquad z=1.$$

Answer: $z=1$.

Example 3. $\frac{a}{x+1}+\frac{b}{(x+1)^2}+\frac{c}{x-1}=\frac{x^2+5x+6}{(x+1)^2(x-1)}$ for which the expression is an identity, find $a; b; c$

.

A) 2; 1; -3 B) 2; -1; -3 C) -2; 1; -3 D) -2; -1; 3

Solution. We bring both sides of the given equation to the same denominator, $(x+1)^2(x-1)$:

$$\frac{a\left(x^2-1\right)+b\left(x-1\right)+c\left(x+1\right)^2}{\left(x+1\right)^2\left(x-1\right)}=\frac{x^2+5x+6}{\left(x+1\right)^2\left(x-1\right)}.$$

From this we obtain the $\left(a+c\right)x^2+\left(b+2c\right)x+\left(-a-c+c\right)=x^2+5x+6$ equality.

Since this equality is an identity, we equate the coefficients of corresponding terms:

$$a+c=1,$$
$$b+2c=5,$$
$$-a-b+c=6$$

Solving this system of equations, we find the andlues of a, b and c $a=-2$, $b=-1$, $c=3$ By the condition of the problem, it follows that $abc=\left(-2\right)\ \left(-1\right)\ 3=6$.

Answer: $abc=6$.

Example 4. $\frac{1}{x}+\frac{1}{y}=\frac{1}{7}$ How many integer solutions does the equation have?

A) 4 B) 5 C) 6 D) 8

Solution. From the given equation, x or y we find the unknown:

$$\frac{1}{x}=\frac{1}{7}-\frac{1}{y}=\frac{y-7}{7y} \qquad x=\frac{7y}{y-7}=\frac{7\left(y-7\right)+49}{y-7}=7+\frac{49}{y-7}$$

By the condition of the problem, x and y For the numbers to be integers, $\frac{49}{y-7}$ the expression must be an integer. It follows that the denominator of the fraction consists of divisors of 49, i.e., $y-7=\left\{\pm1;\ \pm7;\ \pm49\right\}$.

Therefore, x and y the number of admissible integer andlues is 6.

Answer: The equation has 6 integer solutions.

Example 5. $\frac{x-5}{2}=\frac{y-3}{3}=\frac{z+2}{4}$, $2x+4y-3z=32$ if $x+y-z$ find the andlue of

A) 9 B) 10 C) 11 D) 12

Solution. We write the system of equations in the following

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$$6x-30=4y-12=3z+6$$
$$2x+4y-3z=32$$

form. From the first equation, we express $4y$ and $3z$ in terms of x and substitute into the second equation,

$$4y=6x-18, \qquad 3z=6x-36.$$

Substituting into the second equation,

$$2x+(6x-18)-(6x-36)=32$$

we obtain a single-andriable equation. Solving it, $x=7$ we get the andlue. Using the found x , we find y and z we also find:

$$y=\frac{3x-9}{2}=\frac{3\ \ 7-9}{2}=6, \qquad x=7,$$
$$z=2x-12=2\ \ 7-12=2 \qquad y=6,$$
$$z=2.$$

Bundan $x+y-z=7+6-2=11$ we obtain the result.

Answer: $x+y-z=11$.

Example 6. $\begin{cases} 4x-3y=\dfrac{50}{x}, \\ y-x=\dfrac{14}{y} \end{cases}$ if $|2x-y|$ find the andlue of

A) 4 B) 6 C) 8 D) 10

Solution. We transform the given system of equations as follows:

$$\begin{cases} 4x^2-3xy=50, \\ y^2-xy=14 \end{cases} \qquad 4x^2-4xy+y^2=64$$

$$(2x-y)^2=8^2 \qquad |2x-y|=8.$$

Therefore, $|2x-y|=8$.

Answer. $|2x-y|=8$.

Example 7. $\begin{matrix} xyz=36, \\ x-2y=4z \end{matrix}$ if $(4z-x)(x-2y)(2z+y)$ find the andlue of

A) -288 B) -144 C) 144 D) 288

Solution. From the given system of equations, we obtain the following

$$\begin{aligned} 4z-x&=-2y, \\ x-2y&=4z, \\ 2z+y&=\frac{x}{2}, \\ xyz&=36 \end{aligned}$$

relations.

Using the foundlarexpress so‘ralgan $(4z-x)(x-2y)(2z+y)$ expression, we obtain the equality $(4z-x)(x-2y)(2z+y)=(-2y)4z\frac{x}{2}=-4xyz=-4\ \ 36=-144$ equality.

Answer. -144.

Example 8. $\begin{matrix} \frac{7}{x+y}-\frac{6}{x-y}=-1, \\ \frac{1}{x+y}+\frac{1}{x-y}=\frac{10}{21} \end{matrix}$ the system of equations is given, x find

A) -5 B) -2 C) 2 D) 5

Solution. $\frac{1}{x+y}=a$ and $\frac{1}{x-y}=b$ if we introduce the notation, the given system of equations takes the following $\begin{matrix} 7a-6b=-1, \\ a+b=\frac{10}{21} \end{matrix}$ form. Solving the resulting system of equations, a and b we find: $\begin{matrix} 7a-6b=-1, \\ a+b=\frac{10}{21} \end{matrix}$ $13a=\frac{13}{7}$ $a=\frac{1}{7}$.

Using the found a , we find b we find: $b=\frac{10}{21}-a=\frac{10}{21}-\frac{1}{7}=\frac{1}{3}$

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Then, $\begin{array}{l} \dfrac{1}{x+y}=\dfrac{1}{7}, \\ \dfrac{1}{x-y}=\dfrac{1}{3} \end{array}$ $\quad \begin{array}{l} x+y=7, \\ x-y=3 \end{array}$ $\quad \begin{array}{l} x=5, \\ y=2. \end{array}$

Answer. $x=5$.

Example 9. $\begin{array}{l} x_1+x_2+x_3=1, \\ x_2+x_3+x_4=2, \\ x_3+x_4+x_5=3, \\ \dots\ \dots\ \dots \\ x_7+x_8+x_1=7, \\ x_8+x_1+x_2=8 \end{array}$ from the system of equations x_2 find the andlue of

A) -3 B) -1 C) 1 D) 3

Solution. Subtracting the equations in the given system consecutively, we obtain the following

$$\begin{array}{l} x_4-x_1=1, \\ x_5-x_2=1, \\ x_6-x_3=1, \\ x_7-x_4=1, \\ x_8-x_5=1, \\ x_1-x_6=1, \\ x_2-x_7=1 \end{array}$$

system of equations.

From the first equation, $x_4=x_1+1$ from the second equation, $x_7=x_4+1=x_1+2$ and from the last one, $x_2=x_7+1=x_1+3$; and $x_5=x_2+1=x_1+2$; $x_8=x_5+1=x_1+5$; $x_3=x_6-1$ we obtain equalities. But $x_1-x_6=1$ $x_6=x_1-1$.

Therefore, $x_3=x_6-1=x_1-2$.

We substitute the found andlues into the first equation

$x_1+x_2+x_3=1$ $\quad x_1+(x_1+3)+(x_1-2)=1$ $\quad x_1=0$,

$x_2=x_1+3=3$. Answer. $x_2=3$.

Exercises for Independent Practice

1. $|2x-y-1|+\sqrt{2x+4}+(z-2y)^2=0$ from the equality, z find

A) -10 B) -6 C) 6 D) 10

2. $\begin{matrix} 3x+2y+4z=18, \\ x+y+4z=15 \end{matrix}$ if $(4x+3y+8z)(2x+y)$ find the andlue of

A) 33 B)66 C) 99 D) 165

3. $x;\ y$ for natural numbers, $\dfrac{1}{x+y-5}+\dfrac{1}{2x-3y+8}=1$ if the equality holds

if $x-y$ find the andlue of. A. -4 B) -3 C) -2 D) -1

4. $\begin{matrix} \dfrac{ab}{c}=4, \\ \dfrac{bc}{a}=36, \\ \dfrac{ac}{b}=9 \end{matrix}$ if c^2 find the andlue of

A) 9 B) 16 C) 81 D) 324

5. $\begin{matrix} x+xy=\dfrac{5}{4}, \\ y^2+y=\dfrac{15}{4} \end{matrix}$ if the equality holds, $\dfrac{x+5y}{x}$ find the andlue of the expression.

A) -2 B) 8 C) 12 D) 16

6. $\begin{matrix} (a+2)x+4y=14-a, \\ (2a-1)x+(a+1)y=a+8 \end{matrix}$ the system of equations a for what andlue of

has infiexpresstely many solutions.

A) 2 B) 3 C) 2; 3 D) 5

7. $x^2-y^2+6x-2y=76$ find the number of natural solutions of the equation.

A) B) 1 C) 2 D) 6

8. $x=\dfrac{a}{a+3b}$ and $y=\dfrac{b}{3a+b}$ if x express y express in terms of.

A) $\frac{8y+1}{y+1}$ B) $\frac{9y}{1-y}$ C) $\frac{1-y}{8y+1}$ D) $\frac{9y-1}{y+1}$

9. $\begin{array}{l} 3x+5y-4z=6, \\ 2x+4y-2z=3 \end{array}$ if $\frac{x}{y}$ find the andlue of

A) -3 B) $-\frac{1}{3}$ C) 1 D) $\frac{7}{2}$

10. $\begin{array}{l} xy=z+9, \\ xz=y+7, \\ y=4-z \end{array}$ from the system of equations x find

A) 1 B) 3 C) 4 D) 5

Conclusion

It is known that solving systems of equations is one of the most understandable and interesting topics for students. Therefore, it is easy to further increase students' interest in mathematics through this topic. For this purpose, it is even more appropriate to use interesting systems of equations like those above.

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