JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS
VOLUME 5, ISSUE 02
MOPNTHLY JOURNALS
ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

COMPATIBILITY OF ALGEBRAIC AND VISUAL APPROACHES IN SOLVING 3RD AND 4TH ORDER EQUATIONS

Rajabov Azamat
Asia International University
Lecturer at the Department of General Technical Sciences

Abstract

The article analyzes a methodology for teaching third- and fourth-degree algebraic equations in which algebraic methods are employed as the primary instructional approach, while visual tools are used as auxiliary instruments. Students often encounter difficulties when searching for rational roots and applying Horner's scheme in solving higher-degree equations. In the study, algebraic methods are considered the main teaching mechanism, whereas GeoGebra is treated as a tool for verifying and reinforcing algebraic solutions. The proposed step-by-step model emphasizes thorough instruction in algebraic methods at the initial stage, followed by visual verification to consolidate the results. Such an approach serves as an effective means of developing students' functional thinking, enhancing their ability to independently identify errors, and fostering mathematical intuition.

Keywords

3rd order equation, 4th order equation, algebraic methods, Gorner scheme, factorization, GeoGebra, visual approach.

1. INTRODUCTION

1.1. Relevance of the topic

The study of algebraic equations of the 3rd and 4th order is of great importance in modern mathematical education. These equations are not only the main part of theoretical mathematics, but are also widely used in physics, engineering, economics, and other applied sciences. Pedagogical experience shows that students encounter a number of difficulties in mastering higher-order equations. face With the development of modern digital technologies , it has become possible to use visual tools alongside traditional algebraic methods. However, the important issue is to use these tools for the right purpose and in the right order.

1.2. Main problems

First, students encounter the following problems when working with 3rd and 4th order equations:

Secondly, the skills of systematically searching for rational roots are not sufficiently developed. Students use the rational roots theorem "by rote," but do not understand its logical basis.

Third, mistakes are often made in the practical application of the Gorner scheme. Despite the effectiveness of this method, many students have difficulty implementing it correctly.

Fourth, once an algebraic solution is found, there is the problem of verifying its correctness and being confident in the result.

1.3. Research Objective

The primary objective of this study is to develop a methodology for effectively using visual aids as an auxiliary tool for checking and reinforcing solutions, while maintaining the priority of algebraic methods. This approach allows students to :

- To strengthen algebraic thinking;
- To develop mathematical intuition;
- To increase confidence in the correctness of the solution;
- Aimed at forming the ability to independently identify errors.

2. MAIN PART

2.1. Basic algebraic methods

2.1.1. Rational the roots search theorem

Rational the roots search theorem whole with coefficient many of equations possible was all rational roots find for is used . To the theorem according to , if $P(x)=a_nx^n+a_{n-1}x^{n-1}\cdots+a_0$ of the equation / in the form of rational root if she is without p free the divisor of the term (a_0) ,q and head divisor of the coefficient (a_n) It will be .

The essence and application of the theorem:

- **Condition:** The coefficients of the polynomial must be integers.
- **Formula:** Roots $x=\pm\frac{p}{q}$

Example: $2x^3 - x^2 - 7x + 6 = 0$

Free term: 6 → divisors: ±1, ±2, ±3, ±6 Prime coefficient: 2 → divisors: ±1, ±2
Possible rational roots: ±1, ±2, ±3, ±6, ±½, ±3/2

Systematic approach:

1. Listing all possible rational roots
2. Check each by putting it into the equation one by one.
3. Using the first found root via the Gorner scheme

Pedagogical recommendation: Students should complete this process in a step-by-step, orderly manner. A systematic approach, not a random check, is the key to success.

2.1.2. Gorner scheme

Horner scheme many linear to two to be the most effective method to be , too many shortening , too his/her value calculation opportunity gives .

Horner scheme using $\mathbf{P(x) = a_n x^n + a_{n-1} x^{n-1} + ... + a_1 x + a_0}$ is a polynomial **x - c** to to be as follows is done :

x=c	a_n	a_{n-1}	a_{n-2}	...	a_1	a_0
	$b_n = a_n$	$b_{n-1} = b_n \cdot c + a_{n-1}$	$b_{n-2} = b_{n-1} \cdot c + a_{n-2}$	...	$b_1 = b_2 \cdot c + a_1$	$b_0 = b_1 \cdot c + a_0$
	b_n	b_{n-1}	b_{n-2}	...	b_1	b_0

Division: $Q(x) = b_nx^{n-1} + b_{n-1}x^{n-2} + ... + b_1$
Remainder: $R = b_0$
P(x) = (xc)·Q(x) + R
If R=0, then the number x=c is a root of the polynomial P(x).

Important tips :

1. Placing the coefficients in the correct order
2. Carefully perform each calculation
3. Pay attention to the odds sign.
4. : if zero – the root, otherwise – the polynomial x = c value in

2.2. Typical mistakes and how to avoid them

1. Errors in finding rational roots:

- Checking all possible roots in a random manner
- Ignoring fractional roots
- Forgetting the sign when working with negative numbers

Prevention method: Establish a clear schedule, possible was Systematic examination of roots .

2. Errors in the Gorner scheme:

Confusing columns and rows
Arithmetic errors

JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS
VOLUME 5, ISSUE 02, MOPNTHLY JOURNALS
ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

Ignoring the sign in front of the coefficient

Prevention method: Draw a clear diagram, check the results of each action separately.

2.3. Visual inspection using GeoGebra

Once the algebraic solution is fully found, GeoGebra performs the following tasks:

1. Visual verification of the solution The roots found by the algebraic method appear on the graph as the points of intersection of the function with the x-axis. This allows the student to be confident in his work.

2. Graphical estimation of the number of roots

- If the graph crosses the x-axis three times → three real roots
- Crosses once → one real and two complex roots

3. Viewing Estimated Values The graph shows the estimated values of the roots. If the result of the algebraic calculation does not match the graph, this is an error signal.

4. Understanding Function Properties Through Graphing:

- Monotonic intervals
- Extreme points

2.4. A complete practical example

Example 1:

$$P(x)=2x^3-3x^2-11x+6$$

Step 1: Find possible rational roots:

- $a_0=6$ divisors of :$\pm1,\pm2,\pm3,\pm6$
- $a_n=2$ divisors of :$\pm1,\pm2$
- Possible rational roots: $\pm1,\pm2,\pm3,\pm6,\pm1/2,\pm3/2$
- Step 2: Check using the Gorner scheme

  a) $\mathbf{x=1}$we check:

Coefficients	2	-3	-11	6
c=1	2	$(2\times1)+(-3)=-1$	$(-1\times1)+(-11)=-12$	$(-12\times1)+6=-6$
Result	2	-1	-12	-6

$P(1)=-6\neq0$ so $\mathbf{x=1}$ It's not a root.

b) $\mathbf{x=-1}$ we check:

Coefficients	2	-3	-11	6
c=−1	2	$(2\times(-1))+(-3)=-5$	$(-5\times(-1))+(-11)=-6$	$(-6\times(-1))+6=12$
Result	2	-5	-6	12

$P(-1)=12\neq0$so$\mathbf{x=1}$ It's not a root.

c) $x=2$we check:

Coefficients	2	-3	-11	6
c=2	2	$(2\times2)+(-3)=1$	$(1\times2)+(-11)=-9$	$(-9\times2)+6=-12$
Result	2	1	-9	-12

$P(2)=-12\neq0$ so $x=2$ It's not a root.

d) $x=3$ we check:

Coefficients	2	-3	-11	6
c=3	2	$(2\times3)+(-3)=3$	$(3\times3)+(-11)=-2$	$(-2\times3)+6=0$
Result	2	3	-2	0

$P(3)=0;\ \ x=3$ So it's a root.

Step 3: Finding the division from the Gorner diagram: $Q(x)=2x^2+3x-2$

Step 4: Solving the Quadratic Equation

$2x^2+3x-2=0$

$D=9+16=25$

$x=(-3\pm5)/4$

$x_1=\frac{1}{2},\ x_2=-2$

Final solution:

$$P(x)=2(x-3)(x-1/2)(x+2)=(x-3)(2x-1)(x+2)$$

Roots: $x=3, x=\frac{1}{2}, x=-2$

Example 2:

$$P(x)=x^4-2x^3-13x^2+14x+24$$

Step 1: Find possible rational roots

$a_\circ=24\rightarrow\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

- $a_n=1\rightarrow\pm1$
- Possible rational roots : $\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

Step 2: Check

x = 1:

Coefficients	1	-2	-13	14	24
c=1	1	-1	-14	0	24
Result	1	-1	-14	0	24

$P(1)=24\neq0$

x = -1:

Coefficients	1	-2	-13	14	24
$c=-1$	1	-3	-10	24	0
Result	1	-3	-10	24	0

$P(-1)=0$ so $x=-1$ It's a root. Division: $x^3-3x^2-10x+24$

Step 3: Solve the new cubic equation

$$Q(x)=x^3-3x^2-10x+24$$

Finding possible rational roots:

$a_\circ=24$ divisors of $\pm1,\pm2,\pm3,\pm4,\pm6,\pm8,\pm12,\pm24$

- $a_n=1$ the divisors of ±1 x = 2:

Coefficients	1	-3	-10	24

JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS
VOLUME 5, ISSUE 02
MOPNTHLY JOURNALS
ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

c=2	1	-1	-12	0
Result	1	-1	-12	0

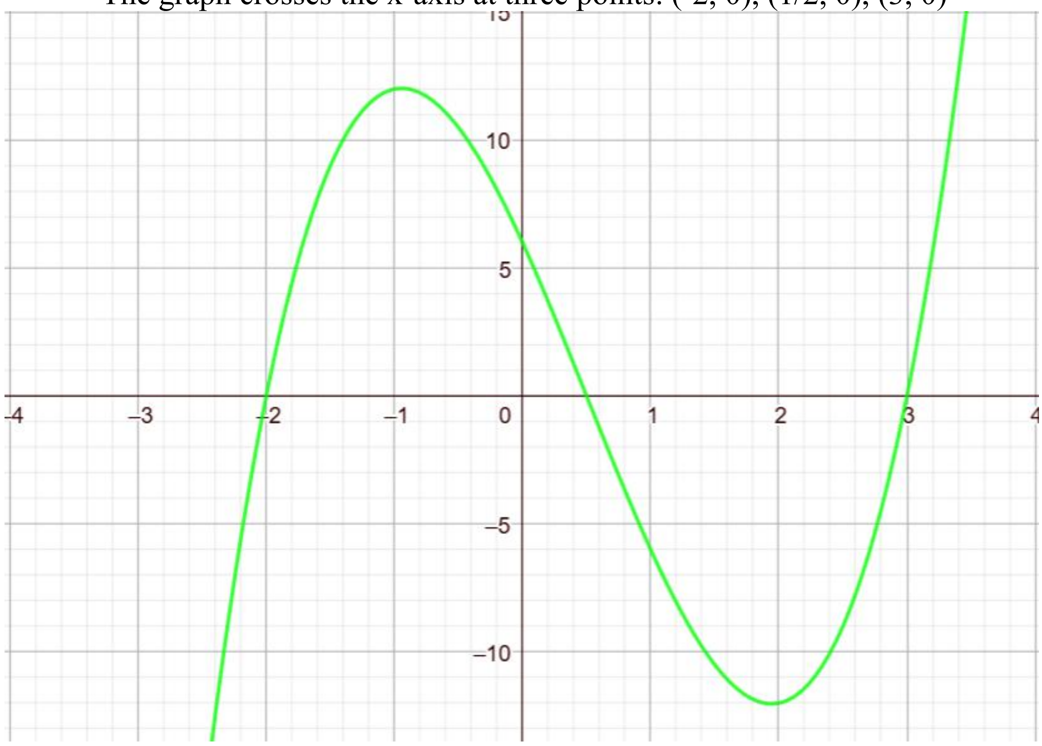
Q(2) =0→x=2root Division:x^2-x-12
Step 4: Quadratic equation

$$x^2-x-12=0$$
$$x=(1\pm\sqrt{(1+48)})/2=(1\pm7)/2$$
$$x=4, x=-3$$

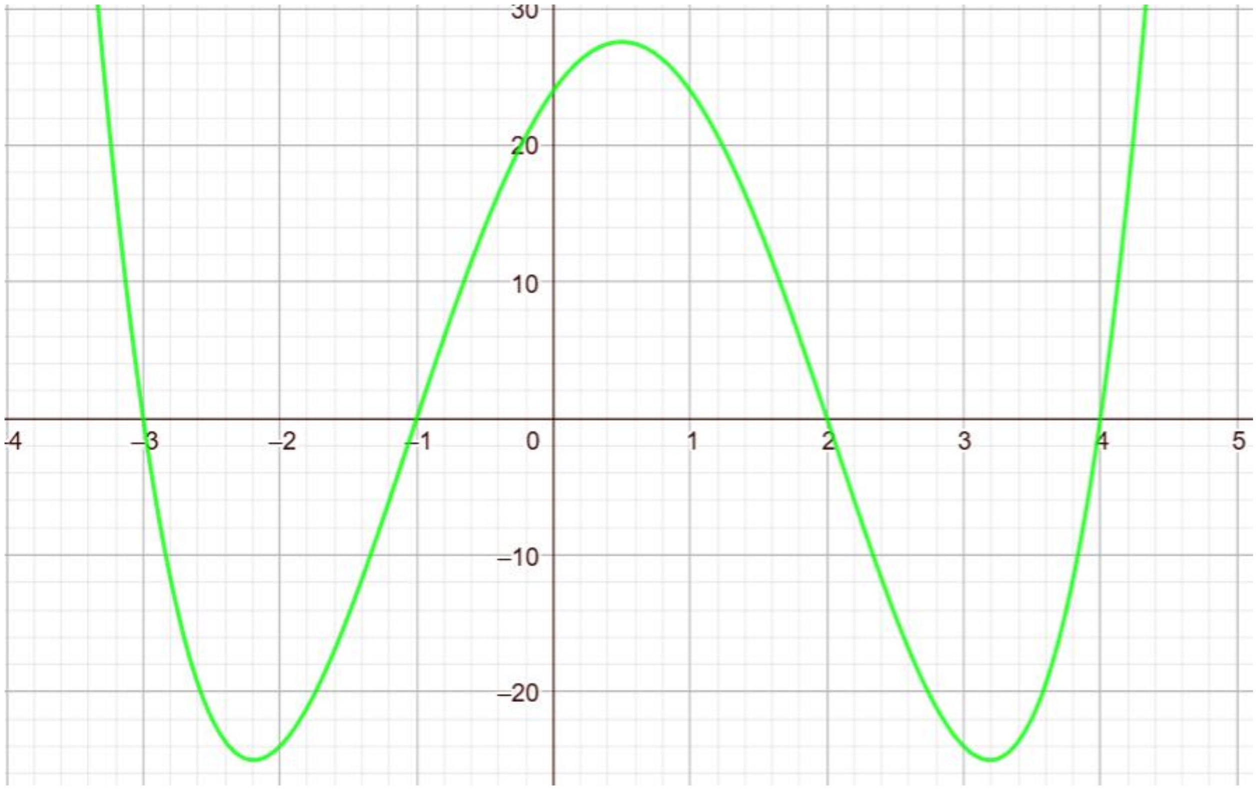
Final solution:

$$P(x)=(x+1)(x-2)(x-4)(x+3)$$

STEP 2: Visual inspection
In GeoGebra
a) $P(x)=2x^3-3x^2-11x+6$ Let's draw the graph of the function:
The graph crosses the x-axis at three points: (-2, 0), (1/2, 0), (3, 0)



b) $P(x)=x^4-2x^3-13x^2+14x+24$ Let's draw the graph of the function:
The graph crosses the x-axis at three points: (-3, 0), (-1, 0), (2, 0), (4, 0)



Conclusion: The reader comes to two important conclusions:
The algebraic method gives accurate and reliable results
The graphic image visually confirms this result and deepens understanding.

Didactic advantages of visual aids

Visual representation transforms abstract algebraic concepts into geometric representations. The student understands that the roots of an equation are not just numbers, but geometric points. The discrepancy between the algebraic and graphical results signals the student to reconsider the error.

Using a graphic image, students:

- Predicting the number of roots
- Separating real and complex roots
- Acquires skills in analyzing function behavior

CONCLUSION

Algebraic methods should remain the main teaching mechanism when teaching 3rd and 4th order equations. These methods form the basis for the formation of mathematical thinking.
Visual tools like GeoGebra should serve a supporting, rather than a primary, function. Their purpose is to confirm the algebraic solution and deepen understanding.
Always prioritize algebraic solutions, visual tools only check in the phase use , initially on paper algebraic solution find , graph the image last in the stage , confirming tool as show to the goal according to .

REFERENCES:

1. Vilenkin N.Ya., Schwartzburd S.I. Algebra and mathematical analysis: Uchebnoe posobie dlya uchashchikhsya school and classes with uglublennym izucheniem mathematiki. Moscow: Prosveshchenie, 2014. 288 p.

JOURNAL OF
MULTIDISCIPLINARY
SCIENCES AND INNOVATIONS

VOLUME 5, ISSUE 02
MOPNTHLY JOURNALS

ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

2. Drijvers P., Doorman M., Boon P., Reed H., Gravemeijer K. The teacher and the tool: instrumental orchestrations in the technology-rich mathematics classroom. Educational Studies in Mathematics, 2010, Vol. 75, No. 2, pp. 213-234.
3. Hohenwarter M., Fuchs K. Combination of dynamic geometry, algebra and calculus in the software system GeoGebra. Computer Algebra Systems and Dynamic Geometry Systems in Mathematics Teaching Conference, 2004, Pecs, Hungary.
4. Mordkovich A.G. Algebra and nachala mathematical analysis. 10-11 klassy: methodicheskoe posobie dlya uchitelya. Moscow: Mnemozina, 2015. 202 p.
5. Preiner J. Introducing Dynamic Mathematics Software to Mathematics Teachers: the Case of GeoGebra. Dissertation, University of Salzburg, 2008. -289 p.