JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS

VOLUME 5, ISSUE 02
MOPNTHLY JOURNALS

ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

DYNAMICS OF STRUCTURAL - INHOMOGENEOUS COAXIAL-MULTILAYERED SYSTEMS "CYLINDER-SHELLS"

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Annotation

The work is devoted to the development of a mathematical model and methodology for assessing the efficiency of dissipative abilities of structurally heterogeneous mechanical systems consisting of multilayer cylinders fastened with a thin viscoelastic shell of finite length. A detailed analysis of known works devoted to this problem is given. A model, methodology and algorithm have been developed for studying the natural and forced vibrations of a system to assess the damping ability of structurally inhomogeneous elastic and viscoelastic mechanical systems, taking into account the influence of geometric and physico-mechanical parameters of the shell and cylinder. In solving the problems considered, the method of divided variables, the method of the theory of potential functions, the Mueller method, the Gauss method and the orthogonal sweep method were used. The complex eigenfrequencies, amplitudes of forced oscillations are determined, and the largest dephasing abilities of the considered neonordic systems are estimated. It was revealed that the effect of large damping abilities in the system manifests itself when the real parts of the complex eigenfrequencies come closer due to the interaction of close eigenmodes with each other.

Keywords: complex natural frequency, damping coefficient, inhomogeneous mechanical system, viscoelasticity, resonant amplitude.

1. Introduction

The intensive development of the national economy leads to the acceleration of scientific and technological progress to the creation of new technological systems that should ensure the efficient operation of apparatuses and machines over a wide range of speeds, temperatures, pressures and loads.

Moreover, they must be efficient for operation, comfortable for humans, environmentally friendly, noiseless, etc. Such technological systems include existing flying machines and various means of helicopters, airplanes, and others. Vibration suppression in such systems is, first of all, arising under the influence of dynamic loads in aircraft and vehicles.

One of the first studies of the problems of the dynamics of shell structures in contact with the cylinder, made of a different material property, was considered in [1–4], which is used today in various machines, devices and structures.

In [5, 6], the problems of oscillations of a thick-walled elastic cylinder (in a flat formulation) under given unsteady loads acting on the external and internal surfaces were investigated. The solution to this problem is mainly based on the use of special functions of mathematical physics. In problems on the propagation of an axisymmetric elastic wave in a cylinder with an elastic filler [7,8], the dispersion equation is obtained. using special functions and Newton's method. the dispersion equation is obtained. The problems of unsteady radial vibrations of a thick-walled elastic cylinder bonded to a thin elastic shell are considered in [8–10], which is solved by the integral transformation method. Such problems are often found in solid fuel rocket engines, where the structural mechanical system consists of cylinders with various dissipative properties.

Along with this, a large number of quasistatic and dynamic problems were solved for axisymmetric bodies under plane deformation. In solving such problems, the quasistatic linear theory of viscoelasticity was usually used. The quasistatic reaction of the system consisting of a cylinder and a housing to the action of a time-varying internal pressure was considered by Blend [9]. Williams et al. [10] also considered several problems of this type. A brief overview of problems for a cylinder with quasistatic viscoelasticity is given in the article by Rogers and Lee [11]. Moreover, the resulting dynamic effects were not taken into account in determining the viscoelastic behavior of the cylinder. However, the occurring vibrations in various elements of the aircraft have led to the need to study the oscillations of visco-elastic cylinders [12,13].

Along with this, a class of problems related to the dynamics of shells in contact with an external elastic or viscoelastic medium of infinite length or limited volume has been considered [14,15]. The problem of the action on the design of short-term pulses initiated, for example, by an explosive, a shock of a solid body, etc., in which the pressure distribution can be localized in the form of a spot of limited size, is considered [16,17]. Experimental studies are limited mainly to recording the final surface parameters of the process, which do not allow us to trace how stress waves develop and interact in the material of structural elements [18, 19, 20]. Using various numerical methods [21,22,23,24] a large number of axisymmetric plane problems were solved in which mechanical systems consist of plates and shells.

In [25, 26, 27], three-dimensional problems of the dynamics of cylindrical bodies were studied using equations of the theory of elasticity under the action of pulsed loads. It is assumed that pressure is applied to the outer surface. The desired components of the displacement vector are presented in the form of a double Fourier series along the longitudinal and angular coordinates.

Along with this study, the dynamics of various heterogeneous systems, taking into account their features and operating conditions, has been the subject of a number of works in which the natural vibrations and dynamic behavior of a structure under various influences are evaluated [37-45].

Here is a review, just some of the works that are devoted to assessing the dynamics of various structures, including axisymmetric systems.

Therefore, the development of an effective methodology and algorithm, as well as the study of the dynamics of heterogeneous axisymmetric systems, is an urgent task.

When solving such problems, it is more difficult to take into account the finite length of multilayer cylindrical bodies fastened in a thin elastic shell [28,29].

This article discusses vibrational processes in spatial structurally inhomogeneous deformable mechanical systems consisting of multilayer cylinders of finite length, the materials of which have both elastic and viscoelastic properties. To assess the dissipative properties of such mechanical systems, mathematically developed models, methods and algorithms, and research results related to providing optimal damping capabilities of the system as a whole are given.

2. Methods

2.1 Models and methods of solution

We consider a viscoelastic multilayer cylinder of finite length of constant internal radius a and c radius of the outer surface b (total radius) along which the cylinder is bonded to a thin elastic (or viscoelastic) shell (Fig. 1). Some of the layers of mechanical systems (multilayer cylinder and shell) can be massless (Fig. 1). In the case under consideration, massless cylindrical bodies are characterized by operator stiffness coefficients, i.e..:

$$\tilde{K}_{ij} f(t) = K_{0ij} \left(f(t) - \int_0^t R_{ij}(t-\tau) f(\tau) d\tau \right), \quad (1)$$

Where K_{0ij} - moments of stiffness massless deformable elements, $R_{ij}(t-\tau)$ - relaxation core, $f(t)$ - arbitrary function of time. The material of the multilayer cylinder is a highly filled polymer, the physicomechanical characteristics of which are determined by the nature of the binder and the adhesion of the binder to the multilayer filler. The considered structurally inhomogeneous mechanical system "cylinder - shell (housing)" is subject to a uniform variable vibrational or pulsed pressure variable in time. The longitudinal section of the cylinder is shown in Fig. 2. The relationship between stresses and strains for structurally inhomogeneous bodies can be represented as

For simplicity, we assume that the edges of the cylinder and the shell can rotate without deformation in the longitudinal direction (Fig. 2). This is equivalent to the fact that the ends of the cylinder are fastened with a flexible membrane, absolutely rigid in its plane:

$$\vartheta = w = -\frac{\partial u}{\partial z} = \frac{\partial^2 u}{\partial z^2} = 0, u_r = u_\varphi = u_z = 0 \quad (z = 0, l).$$

The inner surface is not loaded, i.e..

$$\sigma_r = \tau_{r\varphi} = \tau_{rz} = 0 \quad (r = a). \quad (6)$$

The outer surface may be loaded

$$\sigma_r = -P(\varphi, z, t); \quad \tau_{r\varphi} = \tau_{rz} = 0 \quad (r = b). \quad (7)$$

In the construction (Fig. 1), the left end is rigidly pinched (z = 0), and the right end is free (z=*l*) from loads. In the study of forced oscillations, it is assumed that a local load is applied to the external surface (7).

Now the general problem under consideration can be formulated as follows: it is necessary to find the function $\vec{u}(u_r, u_\varphi, u_z)$ satisfying equations (3) - (4) under boundary conditions (5-6) under the action of a load (7)

2.2 Methods for solving the problem

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In a linear formulation, a solution to the equations of a multilayer cylinder is sought in the form of a Green – Lame expansion [32]

$$u_{rk}=\frac{\partial\Phi_{1k}}{\partial r}+\frac{1}{r}\frac{\partial\Phi_{2k}}{\partial\varphi}+\frac{1}{\nu_{p1}}\frac{\partial^2\Phi_{3k}}{\partial z\partial r},$$
$$u_{\varphi k}=\frac{1}{r}\frac{\partial\Phi_{1k}}{\partial\varphi}-\frac{\partial\Phi_{2k}}{\partial r}+\frac{1}{\nu_{p1}r}\frac{\partial^2\Phi_{3k}}{\partial z\partial\varphi},\qquad(8)$$
$$u_{zk}=\frac{\partial\Phi_{1k}}{\partial z}+\frac{1}{\nu_{p1}}\frac{\partial^2\Phi_{2k}}{\partial z^2}+\nu_{p1}\Phi_{3k},$$

where are the functions Φ_i – the essence of solving scalar wave equations

$$^2\Phi_{1k}=\frac{1}{a_{1k}^2}\frac{\partial^2\Phi_{1k}}{\partial t^2},$$
$$^2(\Phi_{2k},\Phi_{3k})=\frac{1}{a_{2k}^2}\frac{\partial^2\Phi_{1k}}{\partial t^2}.\qquad(9)$$

Here $a_{1k}^2=(\lambda_k+2\mu_k)R_{\lambda,\mu}/\rho_k, a_{2k}^2=\mu_k R_\mu/\rho_k,$

$$R_{\lambda\mu k}=(\lambda_{0k}\Gamma_{\lambda k}^c(\omega_R)+2\mu_{0k}\Gamma_{\mu k}^c(\omega_R))+i(\lambda_{0k}\Gamma_{\lambda k}^s(\omega_R)+2\mu_{0k}\Gamma_{\mu k}^s(\omega_R))$$
$$R_{\mu k}=\mu_{0k}\Gamma_{\mu k}^c(\omega_R)+i\mu_{0k}\Gamma_{\mu k}^s(\omega_R)$$

Solutions of equations (9) can be found depending on whether the wave numbers are positive, zero or negative [33]. At $R_{\lambda\mu k}=R_{\mu k}=0$ solutions of equation (9) are known [34]. Thus, using (8), we determine the displacements of the cylinder points:

$$u_{rk}=_{n=0}\begin{matrix}\gamma_{1k}\left[A_{1n}J_n(\gamma_{1k}r)+A_{2n}Y_n(\gamma_{1k}r)\right]+\frac{n}{r}\left[A_{3n}J_n(\gamma_{2k}r)+A_{4n}Y_n(\gamma_{2k}r)\right]-\\-\frac{\alpha\gamma_{2k}}{\nu_2}\left[A_{5n}J_n(\gamma_{2k}r)+A_{6n}Y_n(\gamma_{2k}r)\right]\end{matrix}\quad\sin(\alpha z+e)\cos n\varphi e^{-i\omega t},$$

$$u_{\varphi k}=-_{n=0}\frac{n}{r}\begin{matrix}\frac{r\gamma_{2k}}{n}\left[A_{3n}J_n(\gamma_{2k}r)+A_{4n}Y_n(\gamma_{2k}r)\right]-\left[A_{1n}J_n(\gamma_{1k}r)+A_{2n}Y_n(\gamma_{1k}r)\right]-\\-\frac{\alpha}{\nu_2}\left[A_{5n}J_n(\gamma_{2k}r)+A_{6n}Y_n(\gamma_{2k}r)\right]\end{matrix}\quad\sin(\alpha z+e)\sin n\varphi e^{-i\omega t},$$

$$u_{zk}=-_{n=0}\frac{\gamma_{2k}^2}{\nu_2}\left[A_{5n}J_n(\gamma_{2k}r)+A_{6n}Y_n(\gamma_{2k}r)\right]+\alpha\left[A_{1n}J_n(\gamma_{1k}r)+A_{2n}Y_n(\gamma_{1k}r)\right]\quad\cos(\alpha z+e)\cos n\varphi e^{-i\omega t}.$$

(10)

Here the prime means differentiation with respect to $\gamma_{1k}r$ or $\gamma_{2k}r$, $J_n(z)$, $Y_n(z)$ - Bessel functions complex argument. The following notation is introduced:

$$\gamma_{1k}^2=\nu_{1k}^2-\alpha_m^2=\omega^2/a_{1k}^2-\alpha_m^2,$$
$$\gamma_{2k}^2=\nu_{2k}^2-\alpha_m^2=\omega^2/a_{2k}^2-\alpha_m^2,\alpha_m=m\pi/l\ (m=1,2,..),$$

Knowing the displacements (10) of various points of structurally inhomogeneous mechanical systems, according to Hooke's law, one can find the components of the stress tensor [34]. Satisfying the boundary and contact conditions (5) - (7), we obtain a system of homogeneous algebraic systems of equations with complex coefficients with nine unknowns

$$\sum_{j=1}^{9} A_j a_{ij} = 0 \quad ,(i = 1,2,3\ldots9) \tag{11}$$

Where

In order for a system of homogeneous algebraic equations with complex coefficients to have solutions, the determinant of this system must be equal to zero. This will give a frequency equation

$$\Delta(\vec{l}, \omega_R, \omega_I) = 0 \quad , \tag{12}$$

Where $\omega = \omega_R + i\omega_I$ - complex frequency, $\vec{l}$ - geometric and physico-mechanical parameters of the mechanical system. The non-trivial solutions (or roots) of complex unified equations (12) are sought numerically by the Mueller method [46].

The real part (ω_R) integrated natural frequency $\omega = \omega_R + i\omega_I$ means the oscillation frequency of the system in question, and the imaginary (ω_I) determines the rate of damping of oscillations and the damping coefficient makes sense. When exposed to a system of external vibrational loads, equations (11) take the following form

$$\sum_{j=1}^{9} A_j a_{ij} = P_j \quad ,(i = 1,2,3\ldots9) . \tag{13}$$

As a result, we obtain a system of inhomogeneous algebraic equations with complex coefficients (13). Here P_j – the amplitude of the vibration effect. This problem is solved by the Gauss methods [45,47] with the selection of the main element.

3. Results and discussions

3.1. Natural oscillations.

A structurally heterogeneous system consisting of a viscoelastic multilayer cylinder with a circular cavity bonded to a thin elastic (or viscoelastic) shell is considered. The relaxation core for deformable viscoelastic cylinders was chosen as the Rzhanitsyn – Koltunov core [28, 30], i.e.:

$$R(t) = Ae^{-\beta t}t^{\alpha-1} \tag{14}$$

Where A, α, β - kernel parameters. The viscous properties of a multilayer dissipatively inhomogeneous cylinder are adopted so that its creep deformation during the quasistatic process amounts to a small fraction (~ 12%) of the total deformation. The procedure for determining the parameters of the relaxation core (14) for various materials is described in [28, 35]. In [36], on the basis of the described methodology, parameters are given for some specific materials.

5. Conclusions

1. A mathematical model and methodology has been developed for evaluating the efficiency of dissipative abilities of structurally inhomogeneous mechanical systems consisting of multilayer cylinders bonded to a thin viscoelastic shell of finite dimensions under various dynamic influences

2. The natural and forced oscillations of structurally homogeneous and inhomogeneous mechanical systems are investigated, the complex natural frequencies and resonant amplitudes of forced oscillations are determined for various values of the instantaneous stiffness of the shock absorber under periodic effects.

3. The greatest dissipative abilities of structurally heterogeneous mechanical systems were revealed when the real parts of complex eigenfrequencies came closer due to the interaction of close eigenmodes, which is not observed in homogeneous systems.

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