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ALGORITHM FOR CONSTRUCTING THE GENERAL SOLUTION OF THE BERNOULLI DIFFERENTIAL EQUATION

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Annotation

The Bernoulli differential equation occupies an important position in the theory of ordinary differential equations due to its nonlinear structure and its reducibility to a linear differential equation through an appropriate transformation. The present study develops a systematic and rigorous algorithm for obtaining the general solution of the Bernoulli differential equation and examines its theoretical foundation, transformation properties, and structural characteristics. The work provides a detailed mathematical framework describing the reduction of nonlinear equations of Bernoulli type to linear equations by means of power transformations and investigates the conditions under which such transformations are valid. Special attention is given to the existence and uniqueness of solutions, exceptional cases, and structural properties of the resulting linear equations. The proposed algorithm is analyzed from both analytical and pedagogical perspectives, emphasizing its applicability in solving nonlinear problems encountered in applied mathematics, mathematical physics, engineering sciences, and mathematical modeling. The study demonstrates that the Bernoulli equation serves as an intermediate class between linear and nonlinear differential equations, illustrating fundamental methods of nonlinear transformation and linearization. The results establish a coherent mathematical procedure for constructing general solutions and clarify the theoretical significance of Bernoulli equations in the qualitative theory of differential equations.

Keywords: Bernoulli differential equation, nonlinear differential equation, linearization method, ordinary differential equations, transformation techniques, integrating factor, existence of solutions, mathematical modeling.

Introduction

The theory of ordinary differential equations constitutes a fundamental branch of mathematical analysis that studies relations involving unknown functions and their derivatives. Differential equations play a central role in describing natural phenomena, engineering processes, and dynamic systems arising in physics, biology, economics, and other scientific disciplines. Among nonlinear first order differential equations, the Bernoulli differential equation occupies a special position because of its distinctive structure and its reducibility to a linear equation by means of an appropriate transformation.

The Bernoulli differential equation represents one of the simplest nonlinear generalizations of a linear differential equation. Despite its nonlinear nature, it admits an exact analytical solution through a systematic transformation procedure. This property makes the Bernoulli equation an essential object of study in the qualitative and analytical theory of differential equations. The equation provides an important example illustrating how nonlinear problems may be transformed into linear ones, thereby demonstrating fundamental ideas of mathematical modeling and solution techniques.

From a historical perspective, the Bernoulli equation was introduced in the works of Jakob Bernoulli in the late seventeenth century during the development of early calculus. Since then, the equation has been widely studied and has served as a classical example in mathematical

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education and research. Its solution method illustrates the use of substitutions, integrating factors, and structural transformations, which are central techniques in differential equations.

The purpose of this study is to present a rigorous algorithm for obtaining the general solution of the Bernoulli differential equation and to provide a systematic mathematical analysis of its theoretical properties. The investigation focuses on the structure of the equation, the transformation that reduces it to a linear equation, and the construction of the general solution. The study also considers exceptional cases, domains of validity, and the relationship between Bernoulli equations and linear differential equations.

The research aims to provide a comprehensive mathematical description of the solution process while maintaining clarity and theoretical rigor. The results contribute to a deeper understanding of nonlinear transformation methods and provide a useful framework for solving practical problems involving nonlinear differential equations.

Mathematical formulation of the Bernoulli differential equation

Mathematical formulation of the Bernoulli differential equation

$$\frac{dy}{dx} + P(x)y = Q(x)y^n,$$

where the functions *P(x)* and *Q(x)* are continuous on a given interval and the exponent *n* is a real number distinct from zero and one. The condition $n\neq 0,1$ ensures that the equation remains nonlinear and cannot be reduced directly to a standard linear differential equation.

The equation represents a nonlinear extension of the linear first order differential equation. When the exponent equals zero or one, the equation reduces to a linear form. Therefore, the essential mathematical interest arises when the exponent differs from these values.

The nonlinear term y^n introduces additional complexity into the equation. However, the structure of the equation allows the use of a power transformation that converts the equation into a linear differential equation with respect to a new dependent variable.

Theoretical foundation of the transformation method

The application of arithmetic progressions in real-life processes is evident in several fields:

The fundamental idea underlying the solution of the Bernoulli equation consists in introducing a transformation that removes the nonlinear term. Consider the substitution

$$z = y^{1-n}.$$

Assuming that the function *y(x)* is nonzero and differentiable, the derivative of the new variable is obtained using the chain rule:

$$\frac{dz}{dx} = (1-n)y^{-n}\frac{dy}{dx}.$$

Multiplying the original equation by y^{-n} yields.

$$y^{-n}\frac{dy}{dx} + P(x)y^{1-n} = Q(x).$$

Using the transformation $z=y^{1-n}$, the equation becomes

$$\frac{1}{1-n}\frac{dz}{dx} + P(x)z = Q(x).$$

Multiplying both sides by 1−n gives the linear differential equation

$$\frac{dz}{dx} + (1-n)P(x)z = (1-n)Q(x).$$

Thus, the nonlinear Bernoulli equation is reduced to a linear equation in the variable z. This transformation establishes the key property of Bernoulli equations, namely their reducibility to linear equations.

Algorithm for obtaining the general solution

The solution procedure for the Bernoulli differential equation may be formulated as a systematic algorithm consisting of several successive steps.

The equation is first written in standard form by isolating the derivative and identifying the functions P(x), Q(x) and the exponent nnn. This normalization ensures that the equation matches the canonical Bernoulli structure.

The power substitution $z=y^{1-n}$ is then introduced. The derivative of this expression is computed using the chain rule, allowing the original equation to be expressed in terms of the new variable.

After substitution, the equation becomes linear in the variable z. The resulting linear differential equation is solved using the integrating factor method. If the linear equation has the form

$$z' + a(x)z = b(x),$$

the ntegrating factor is defined by

$$\mu(x) = \exp\left(\int a(x)\,dx\right).$$

Multiplying the equation by the integrating factor transforms the left side into the derivative of a product, which allows direct integration. The general solution for z(x) is obtained by integrating both sides.

Finally, the original variable y(x) is recovered from the relation $z=y^{1-n}$. This produces the general solution of the Bernoulli equation.

Existence and uniqueness of solutions

The existence and uniqueness of solutions for the Bernoulli equation follow from general theorems for first order differential equations. If the functions P(x) and Q(x) are continuous on an interval and the initial value problem is well defined, then a unique solution exists locally.

However, the transformation $z=y^{1-n}$ requires that the function y(x) does not vanish when nnn is not an integer. This condition imposes restrictions on the domain of the solution and must be considered in the analysis.

Exceptional and special cases

Several special cases arise in the study of Bernoulli equations. When the exponent equals zero, the equation becomes a linear differential equation. When the exponent equals one, the equation reduces to a separable equation. These cases require separate treatment and are not considered genuine Bernoulli equations.

Another special situation occurs when the function Q(x) vanishes identically. In this case the equation reduces to a homogeneous linear differential equation, which may be solved directly without transformation.

If the exponent is a fractional number, the solution may require additional restrictions on the sign of the function to ensure that the transformation remains defined. These considerations illustrate the importance of domain analysis in nonlinear differential equations.

Structural properties of Bernoulli equations

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Bernoulli equations possess several structural properties that distinguish them from other nonlinear equations. The equation exhibits a multiplicative nonlinear term that depends on a power of the dependent variable. This structure allows a power transformation to eliminate the nonlinearity.

The transformation also demonstrates an important principle in the theory of differential equations, namely that nonlinear problems may sometimes be solved by converting them into linear problems through appropriate changes of variables.

The equation therefore serves as a prototype for more general nonlinear equations that admit linearizing transformations.

Analytical interpretation of the general solution

The general solution of the Bernoulli equation reflects the combined influence of the linear and nonlinear components of the equation. The integrating factor accounts for the linear part, while the power transformation handles the nonlinear term.

The resulting solution typically involves exponential functions, power functions, and integrals of the coefficient functions. The structure of the solution provides insight into the behavior of solutions and their dependence on initial conditions.

Conclusion

The Bernoulli differential equation represents an important class of nonlinear first order differential equations characterized by its reducibility to a linear equation through a power transformation. The study has presented a systematic algorithm for obtaining the general solution and has analyzed the theoretical foundation of the transformation method.

The investigation has demonstrated that the Bernoulli equation illustrates fundamental principles of nonlinear analysis, including transformation techniques, existence of solutions, and structural properties of differential equations. The equation provides a clear example of how nonlinear problems may be approached through linearization methods.

The results confirm the mathematical significance of Bernoulli equations in the theory of ordinary differential equations and highlight their importance in both theoretical and applied contexts. The developed algorithm offers a reliable method for constructing general solutions and contributes to a deeper understanding of nonlinear differential equations.

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