JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS

VOLUME 5, ISSUE 02
MOPNTHLY JOURNALS

ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

METHODOLOGY FOR SIMPLIFYING PERFECT NORMAL FORMS IN BOOLEAN ALGEBRA

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Abstract. This article explains the methodology for simplifying perfect normal forms in Boolean algebra through specific examples in the process of teaching the subject "Discrete Structures" to students of the "Artificial Intelligence" educational program at technical higher education institutions.

Keywords: Higher education, artificial intelligence, discrete structures, Boolean algebra, logical functions, perfect normal form, perfect disjunctive normal form, perfect conjunctive normal form, simplification, algorithm.

Introduction

In order to contribute, even to a small extent, to the implementation of the tasks defined in the Resolution of the President of the Republic of Uzbekistan Shavkat Mirziyoyev No. PQ-3775 dated June 5, 2018 "On additional measures to improve the quality of education in higher education institutions and ensure their active participation in comprehensive reforms being implemented in the country", the Resolution No. PQ-4708 dated May 7, 2020 "On measures to improve the quality of education in mathematics and develop scientific research", and the Resolution No. PQ-4996 dated February 17, 2021 "On measures to create conditions for the accelerated implementation of artificial intelligence technologies", as well as other regulatory legal documents, we considered it appropriate to write this article [1, 2, 3].

Boolean algebra is one of the important branches of discrete mathematics and is widely used in information technology, programming, artificial intelligence, automation, and the theory of digital devices. In particular, the analysis and simplification of logical functions is of great practical importance. Logical functions are often given in perfect normal forms—Perfect Disjunctive Normal Form (PDNF) or Perfect Conjunctive Normal Form (PCNF). However, such forms are often complex and contain redundant elements. Therefore, simplifying them is a relevant task.

This article analyzes the concepts of Perfect Disjunctive Normal Form (PDNF) and Perfect Conjunctive Normal Form (PCNF), their mathematical foundations, and methods of transforming these forms into simpler expressions in the process of teaching the subject "Discrete Structures" to students of the "Artificial Intelligence" program at technical higher education institutions.

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Literature Review

The problem of simplifying perfect normal forms in Boolean algebra has been discussed in many local and international studies. One of the fundamental sources in this field is K. Rosen's book "Discrete Mathematics and Its Applications", where logical functions, normal forms, and the theoretical foundations of their minimization are described in detail. In the book "Discrete and Combinatorial Mathematics" by R. Grimaldi, Boolean algebra is presented in close connection with combinatorics. In addition, the problem of simplifying logical functions is enriched by algorithmic approaches. Among Uzbek researchers, Anvar Kabulov, Mansur Berdimurodov, and Abdussattar Baizhumanov wrote an article on the methodology of simplifying logical normal forms, specifically on the topic "Minimization of k-valued logical functions in the class of disjunctive normal forms" [DOI: https://doi.org/10.26577/JMMCS202412114].

In the author's own textbook "Discrete Structures", published in 2025, theoretical and practical information on normal forms and their analysis in Boolean algebra is provided. In recent years, scientific articles have paid special attention to the use of computers, algebra systems, and software tools in simplifying logical functions. This enables integration with modern information technologies.

Methodological Approach

When simplifying perfect normal forms, the following issues must be considered:

- The given expression (formula) should not be excessively large;
- The number of redundant logical elements in practical circuits (truth tables) should not increase;
- The computation and analysis process should not become overly complicated.

The algebraic simplification method of perfect normal forms is based on the basic laws and properties of Boolean algebra. This approach is convenient for small-scale expressions and is suitable for manual calculation.

Method and Results

Basic Concepts: To make all reasoning easier to study, it is possible to transform expressions into a general standard form using logical laws.

Definition 1. A disjunction of conjunctive monomials is called a Disjunctive Normal Form (DNF) [11].

Definition 2. A conjunction of disjunctive monomials is called a Conjunctive Normal Form (CNF) [11].

Definition 3. If in a monomial only one of the pair $A_i$or $\bar{A}_i$is present, then conjunctive or disjunctive monomials of the propositional variables $A_1,A_2,...,A_n$are called perfect [11].

Definition 4. If a CNF contains non-repeating perfect disjunctive monomials of the propositional variables $A_1,A_2,...,A_n$, then it is called a Perfect Conjunctive Normal Form (PCNF) [11].

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Definition 5. If a DNF contains non-repeating perfect conjunctive monomials of the propositional variables $A_1,A_2,\ldots,A_n$, then it is called a Perfect Disjunctive Normal Form (PDNF) [11].

Example

Let us transform the formula

$$\alpha=(A\leftrightarrow(B\wedge C))\rightarrow(\bar{B}\leftrightarrow A)$$

into PDNF and PCNF [11].

Solution.

We use the following basic logical equivalences:

$$x\rightarrow y\equiv\bar{x}\vee y$$
$$x\leftrightarrow y\equiv(x\wedge y)\vee(\bar{x}\wedge\bar{y})$$

Let us expand the left and right parts of the given expression.

Left part:

$$A\leftrightarrow(B\wedge C)=(A\wedge B\wedge C)\vee(\bar{A}\wedge(B\bar{\wedge}C))$$
$$(B\bar{\wedge}C)=\bar{B}\vee\bar{C}$$

Right part:

$$\bar{B}\leftrightarrow A=(\bar{B}\wedge A)\vee(B\wedge\bar{A})$$

Implication:

$$\alpha=(A\leftrightarrow(\bar{B}\wedge C))\vee(\bar{B}\leftrightarrow A)$$

We construct the truth table (Table 1):

Table 1.

A	B	C	α	Elementary conjunctions for rows where α=1	Elementary disjunction representing cases where α=0
0	0	0	1	$\overline{ABC}$	
0	0	1	1	$\overline{AB}C$	
0	1	0	1	$\bar{A}B\bar{C}$	
0	1	1	0		$A\vee\bar{B}\vee\bar{C}$
1	0	0	0		$\bar{A}\vee B\vee C$
1	0	1	0		$\bar{A}\vee B\vee\bar{C}$

1	1	0	1	$AB\bar{C}$	
1	1	1	1	ABC	

Note:

$$A\wedge B\wedge C=A\cdot B\cdot C=ABC$$

Thus, the final answer is:

$$\alpha=A\bar{B}C\vee\bar{A}BC\vee\bar{A}B\bar{C}\vee AB\bar{C}\vee ABC$$

This is the PDNF.

$$\alpha=(A\vee\bar{B}\vee\bar{C})\wedge(\bar{A}\vee B\vee C)\wedge(\bar{A}\vee B\vee\bar{C})$$

This is the PCNF.

Simplification of the PCNF:

Now we simplify the PCNF.

We apply the absorption rule and consider the following two parentheses:

$$(\bar{A}\vee B\vee C)\text{and}(\bar{A}\vee B\vee\bar{C})$$

We apply the rule:

$$(x\vee C)\wedge(x\vee\bar{C})\equiv x$$

Let $x=(\bar{A}\vee B)$. Then:

$$(\bar{A}\vee B\vee C)\wedge(\bar{A}\vee B\vee\bar{C})=(\bar{A}\vee B)$$

Therefore, the simplified PCNF can be written as:

$$\alpha=(A\vee\bar{B}\vee\bar{C})\wedge(\bar{A}\vee B)$$

This is the most compact conjunctive normal form. Simplifying perfect normal forms develops students' logical thinking, forms algorithmic reasoning, and helps them deeply master the basics of digital circuits and programming.

Conclusion

Perfect normal forms in Boolean algebra serve as an important theoretical basis for expressing logical functions. However, in practical problems, it is necessary to simplify them. Minimizing perfect normal forms using algebraic, graphical, and algorithmic methods makes it

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possible to transform logical expressions into efficient and convenient forms. These approaches are important not only practically but also didactically.

Recommendation

In the process of teaching the subject "Discrete Structures" to students of the "Artificial Intelligence" program at technical higher education institutions, the effective use of information technologies, algebra systems, and programming languages for simplifying perfect normal forms in Boolean algebra increases learning effectiveness.

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