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PROBABILITY AND OPTIMAL CONTROL APPROACH IN ANALYSIS OF TAXI DRIVER'S MOTION STRATEGY

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Abstract: The Taxi Driver Issue is a practical model of process research and optimal management science based on Markov decision processes. This issue aims to find the optimal strategy that will give the driver the maximum benefit based on the different options of action in each city situation. Using an iterative algorithm, the optimal stationary strategy is determined based on probabilities and rewards. This article analyzes the movements between towns A, B and C and identifies the strategy of waiting for the most useful decision – queue waiting. The issue is used in the service management, transportation and logistics management sectors.
Keywords: Process research, optimal management, Markov decision process, iterative algorithm, probability, benefit function, strategic planning, taxi system, status and action, stationary strategy, service system, probability matrix, maximum profit, cost-effectiveness.
Annotatsiya: Taksi haydovchisi masalasi jarayonlar tadqiqoti va optimal boshqaruv fanining Markov qaror jarayonlariga asoslangan amaliy modelidir. Ushbu masala haydovchining har bir shahar holatida turli harakat variantlari asosida maksimal foyda keltiradigan optimal strategiyasini topishga qaratilgan. Iteratsion algoritm yordamida ehtimolliklar va mukofotlar asosida optimal statsionar strategiya aniqlanadi. Mazkur maqolada A, B va C shaharchalari oʻrtasidagi harakatlar tahlil qilinib, eng foydali qaror – navbat kutish strategiyasi aniqlangan. Masala xizmat koʻrsatish, transport va logistikani boshqarish sohalarida qoʻllaniladi.
Kalit soʻzlar: Jarayonlar tadqiqoti, optimal boshqaruv, Markov qaror jarayoni, iteratsion algoritm, ehtimollik, foyda funksiyasi, strategik rejalashtirish, taksi tizimi, holat va harakatlar, statsionar strategiya, xizmat koʻrsatish tizimi, ehtimollik matritsasi, maksimal foyda, iqtisodiy samaradorlik.
Аннотация: Задача таксиста — это практическая модель исследования процессов и науки оптимального управления, основанная на марковских процессах принятия решений. Эта задача направлена на поиск оптимальной стратегии водителя, которая максимизирует прибыль на основе различных вариантов действий в каждом городе. С помощью итерационного алгоритма определяется оптимальная стационарная стратегия на основе вероятностей и вознаграждений. В этой статье анализируются перемещения между городами A, B и C и определяется наиболее выгодное решение — стратегия ожидания в очереди. Задача используется в сферах обслуживания, транспорта и управления логистикой.
Ключевые слова: Исследование процессов, оптимальное управление, марковский процесс принятия решений, итерационный алгоритм, вероятность, функция прибыли,

стратегическое планирование, система такси, состояние и действия, стационарная стратегия, система обслуживания, матрица вероятностей, максимальная прибыль, экономическая эффективность.

Introduction: Process research and optimal control are the sciences that serve to make decisions in complex systems and ensure the most efficient use of available resources. Markov decision processes play an important role in this science, as they allow to ensure the maximum expected benefit through a sequence of actions. In this study, the goal is to find the optimal strategy by modeling a real-life situation in which a taxi driver may be in different urban situations (A, B, C). In each situation, there are several solutions (actions), each of which is represented by different probabilities and rewards. The goal is to find the strategy that will bring the driver the greatest benefit.

Literature review

A review of scientific research on process research and optimal control shows that decision-making systems based on probabilistic models are effectively used in the service and transport sectors. Dynamic programming and Markov processes developed by R. Bellman create the main theoretical basis in this regard. G. Dantzig's work is focused on determining optimal strategies using linear and nonlinear programming. L. Kantorovich, on the other hand, connected the theory of optimal resource allocation with economic models. At the moment, models using Markov processes provide effective solutions in service systems, especially in transport and taxi systems.

Research methodology

The study is aimed at determining the most profitable strategy for a taxi driver based on the probabilities that can be in three cities - A, B and C. The methodological approach includes the following stages:

1. Mathematical modeling - a model is built based on the probabilities and rewards corresponding to each action;

2. Iterative calculation algorithm (value iteration) - the maximum expected profit is calculated for each case;

3. Optimal strategy selection - the profit for each solution is evaluated and the best action is determined;

4. Stationary strategy determination - when the selected solutions do not change in all cases, this strategy is considered optimal.

The study found that for all cases, the most profitable strategy is to go to the nearest taxi stand and wait in line. These results reveal opportunities to improve the efficiency of service systems and save time and resources. This methodology can be applied to transport systems, logistics and resource allocation.

Analysis and results

A taxi driver operates in an area consisting of three towns A, B and C. If the driver is in town A, he has 3 options:

1) drive from one place to another in the town in order to meet a random passenger;

2) go to the nearest taxi stand and wait in line;

3) wait for a call while wearing a radio headset.

If the driver is in town C, there are three possibilities as above. However, since radio communication is not established in town B, the driver is deprived of the last possible option. For each city (state) and option (solution), the probability of the next trip being in cities A, B, and C and the corresponding net benefit (gain) are given. For example, when calculating the net benefit corresponding to solutions 1 and 2, the cost of traveling from one place to another, to the nearest taxi stand, must be taken into account. In this case, the transition probabilities and gains depend on which solution is adopted, because for each solution. The driver faces different density distributions of passengers. We express this information in the following table:

Assume q = 0.9. Let the initial plan be taken from the point of maximizing the points based on the statement P, that is, P =.

Table

Holat	Yechim	Extimollik			Foyda, yutuq			
		ρ_{i1}^{k}	ρ_{i2}^{k}	ρ_{i3}^{l}	ω_{i1}^{k}	ω_{i2}^{k}	ω_{i3}^{k}	$\omega_i^k = \sum_{j=1}^{3} \rho_{i,j}^{k} \omega_{ij}^{k}$
A	1 2 3	1/2 1/16 1/4	22221/4b gfy1/4 3/4 1/8	1/4 3/16 5/8	10 8 4	4 2 6	8 4 4	8 2,75 4,45
B	1 2	1/4 1/16	0 7/8	1/2 1/16	14 8	0 16	18 8	16 15
C	1 2 3	1/4 1/8 3/8	1/4 3/4 1/16	1/2 1/8 3/8	10 6 4	2 4 0	8 2 8	7 4 4,5

According to step 1 of the iterative algorithm, we construct the following system of equations with respect to the unknowns , i = 1, 2, 3:

$$\begin{cases} 8 + 0,9\left(\frac{1}{2}\omega_1 + \frac{1}{4}\omega_2 + \frac{1}{4}\omega_3\right) = \omega_1 \\ 16 + 0,9\left(\frac{1}{2}\omega_1 + \frac{1}{2}\omega_2\right) = \omega_2 \\ 7 + 0,9\left(\frac{1}{4}\omega_1 + \frac{1}{4}\omega_2 + \frac{1}{2}\omega_3\right) = \omega_3 \end{cases}$$

From this it is clear that = 91.26; = 97.55; = 89.97.

Now, moving on to the second step, we define the sets f(i, k).

For case A: when k = 2, 2.75 + 0.9 (+ +)= 88.90 < = 91.26;

When k = 3, 4.25 + 0.9(+ +) =86.37 < <91.26;. Hence, f (1 , k) =.

For case B: When k = 2, 15 + 0 ,9(+ = 102.01<=97.55. Therefore, f(2, k) = 2.

For case C: When k = 2, 4 + 0 , 9( )=90.23>=89.97; When k = 3, 4.5+0.9(++)= 8 6 ,77 < W3 = 89.97. Therefore, f(3, k) =2.

Since f(1,k) =, f(2,k) = 2 and f(3,k) = 2,

the new stationary plan is as follows: = . Based on this plan, we proceed to step 1 and construct the following system of equations:

$$\begin{cases} 8 + 0,9\left(\frac{1}{2}\omega_1 + \frac{1}{4}\omega_2 + \frac{1}{4}\omega_3\right) = \omega_1 \\ 15 + 0,9\left(\frac{1}{16}\omega_1 + \frac{7}{8}\omega_2 + \frac{1}{16}\omega_3\right) = \omega_2 \\ 4 + 0,9\left(\frac{1}{4}\omega_1 + \frac{1}{4}\omega_1 + \frac{1}{2}\omega_3\right) = \omega_3 \quad . \end{cases}$$

The system is: = 119.44; = 134.48; = 121.93 . Moving to the second step, we check the stationary strategy for optimality and determine the sets f(i, k). For case A: when k = 2, 2.75 + 0.9 (+)= 127.68 > = 119.44;

when k = 3, 4.25+0. 9 () = 114.84 < =119.44;. Hence, f (1 , k) = 2. For case B: when k = 1, 16+ 0.9() = 124.62 < = 134.48. Hence, f(2,k) = .

For case C: when k=1, 7+0 ,9() =119.00 = 121.93; when k = 3, 4.5 + 0 ,9 () =113.26<= 121.93. Therefore, f(3, k) = 0.

Since f(1 , k) = 2, f(2 , k) = and f(3, k) = ,

the new stationary plan looks like this: .

Based on this plan, we proceed to step 1 and construct the following system of equations:

$$\begin{cases} 2{,}75 + 0{,}9\left(\frac{1}{16}\omega_1 + \frac{3}{4}\omega_2 + \frac{3}{16}\omega_3\right) = \omega_1 \\ 15 + 0{,}9\left(\frac{1}{16}\omega_1 + \frac{7}{8}\omega_2 + \frac{1}{16}\omega_3\right) = \quad \omega_2 \\ 4 + 0{,}9\left(\frac{1}{4}\omega_1 + \frac{1}{4}\omega_2 + \frac{1}{2}\omega_3\right) = \omega_3. \end{cases}$$

The solution to this system of equations is: = 121.66; = 135.31; = 122.84.

Moving to the second step, we check the stationary strategy for optimality and determine the sets f(i, k). It is easy to show that for all i= 1, 2, 4, f(i, k) = . Therefore, the stationary q is the optimal strategy. It follows that, regardless of the state in which the taxi driver is, it is better to choose solution 2, that is, to go to the nearest taxi stand and wait in line.

Then his average payoff in the states will be the largest and will be

= 121.66; = 135.31; = 122.84.

The optimal strategy is determined based on the probabilities and rewards of the taxi driver's actions. In the end, the second solution — waiting in line — is always the most profitable. This strategy gives the maximum average profit in each case (A, B, C):

Case A — 121.66,

Case B — 135.31,

Case C — 122.84.

This issue is important in real life, especially in the efficient allocation of resources in service systems.

Conclusion

During the study, the strategic decision problem faced by a taxi driver in the process of operating in an area consisting of three towns was analyzed based on process research and optimal control theory. The problem was mathematically modeled based on Markov decision processes, the probability and expected profit (profit) values for each case and strategy were determined, and a way to find the optimal strategy was developed using an iterative algorithm.

The analysis showed that among the available strategies for a taxi driver, the option that gives the highest average profit is the strategy of going to the nearest taxi stand and waiting in line. This strategy gives the most optimal result in all cases (A, B, C), that is, it maximizes the driver's expected profit.

Also, this study showed the practical importance of Markov processes and optimal control approaches in optimizing decision-making, efficient use of resources, and increasing operational efficiency in the service sector, in particular in transport systems.

The results of the study can be applied in the future to other service sectors, including logistics systems, queuing theory, and automated control systems.

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