



**OPTIMIZATION APPROACHES AND APPLICATION OF MATHEMATICAL
MODEL OF THE TRAVELING SALESMAN PROBLEM IN TOURISM COMPANIES**

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Annotation: This article analyzes the Travelling Salesman Problem (TSP) and its optimization approaches, which play a significant role in the efficient operation of tourism companies. In particular, factors such as planning travel routes, reducing transportation costs, and saving time increase the relevance of this problem. The paper presents the classical formulation of the TSP and examines its mathematical model and combinatorial optimization methods. In particular, approaches such as dynamic programming, genetic algorithms, and approximation algorithms are analyzed, with a focus on their applicability to the activities of tourism companies, supported by practical examples. In addition, the article provides the results of a software simulation based on real-world data from a tourism company and demonstrates the effectiveness of modern algorithms in determining optimal routes. The results of the study can be beneficial in increasing competitiveness in the tourism sector, improving logistics systems, and enhancing the quality of service delivery.

Аннотация: В данной статье рассматривается задача коммивояжера (Travelling Salesman Problem, TSP) и подходы к её оптимизации, которые играют важную роль в эффективной деятельности туристических компаний. Особенно актуальной эта проблема становится при планировании туристических маршрутов, снижении транспортных расходов и экономии времени. В статье приводится классическая формулировка задачи TSP, а также рассматриваются математическая модель и методы комбинаторной оптимизации. В частности, анализируются такие подходы, как динамическое программирование, генетические алгоритмы и аппроксимационные алгоритмы, а также их применение в деятельности туристических фирм на основе практических примеров. Кроме того, в работе представлены результаты программной симуляции, разработанной на основе реальных данных туристической компании, что позволяет продемонстрировать эффективность современных алгоритмов при выборе оптимальных маршрутов. Результаты исследования могут быть полезны для повышения конкурентоспособности в туристической отрасли, совершенствования логистических систем и улучшения качества предоставляемых услуг.

Annotatsiya: Mazkur maqolada turizm firmalarining samarali faoliyat yuritishida muhim ahamiyatga ega bo'lgan kommivoyajyor masalasi (travelling salesman problem, TSP) va uning optimallashtirish yondashuvlari tahlil qilinadi. Ayniqsa, sayyohlik yo'nalishlarini rejalashtirish, transport xarajatlarini kamaytirish va vaqtni tejash kabi omillar bu masalaning dolzarbligini oshiradi.

Maqolada TSP masalasining klassik formulasi keltirilib, uning matematik modeli va kombinator

optimallashtirish usullari asosida yechim usullari ko‘rib chiqiladi. Shu jumladan, dinamik dasturlash, genetika algoritmi, yaqinlashtirish algoritmlari kabi yondashuvlar tahlil qilinib, ularning turizm firmalari faoliyatiga qo‘llanish imkoniyatlari amaliy misollar orqali yoritiladi. Shuningdek, maqolada real hayotdagi turizm firmasi ma'lumotlari asosida ishlab chiqilgan matematik modelning dasturiy tajribasi (simulyatsiyasi) natijalari taqdim etiladi va u orqali optimal yo‘nalishlar aniqlashda zamonaviy algoritmlarning samaradorligi asoslanadi. Maqola natijalari turizm sohasida raqobatbardoshlikni oshirish, logistika tizimlarini takomillashtirish va xizmat ko‘rsatish sifatini yuksaltirishda foydali bo‘lishi mumkin.

Keywords: Travelling Salesman Problem (TSP), Optimization, Combinatorics, Optimal routes, Tourism companies, Mathematical model, Travel routes, Time saving, Algorithm efficiency.

Ключевые слова: Задача коммивояжера (TSP), Оптимизация, Комбинаторика, Оптимальные маршруты, Туристические компании, Математическая модель, Туристические маршруты, Экономия времени, Эффективность алгоритмов.

Kalit so‘zlar: Kommivoyajyor masalasi (TSP), optimallashtirish, kombinatorika, optimal yo‘nalishlar, turizm firmalari, matematik model, sayyohlik yo‘nalishlari, vaqtni tejash, algoritm samaradorligi.

Introduction. The Traveling Salesman Problem (TSP) is a popular and research-intensive problem in combinatorial optimization. Its main goal is to find the shortest route that will take you from one city to another and return to your starting point. This problem is used in many areas, including transportation logistics, manufacturing process optimization, electronic circuit design, and computer network planning. The TSP is, in fact, of great importance in finding solutions to real-world problems and creating efficient systems. Optimization approaches to this problem also play a significant role in the organization of tourism companies and travel services, as the need to plan tourist routes, reduce costs and save time is increasing day by day. There are various methods for solving it, such as: Bruteforce (Full Review), dynamic programming (Held-Karp algorithm), Greedy algorithm (Greedy Algorithm), genetic algorithm, simulated annealing (Simulated Annealing), linear programming and chess (Branch and Bound)

The goal of the traveling salesman problem is to find the shortest and most efficient route for a salesman to visit a given number of cities. Each city must be visited only once, and at the end, the salesman must return to the starting city. The main goal of the problem is to minimize the distance traveled to the cities, thereby reducing time and cost. Solving the TSP, by optimizing the objective, increases efficiency in real-world applications such as transportation, logistics, and resource management. The algorithms and methods developed for solving the TSP can also be applied to many other optimization problems.

Find a cycle that can traverse the given nine cities in the shortest time (distance, cost) by visiting each of them only once. In this case, the number of cycles is at most $n!$. This problem is related to the problem of finding a Hamilton cycle of minimum length. The "Networks and Boundaries" method can be used to solve the traveling salesman problem. This method is carried out using a graph with no cycles and a path, and by constructing tables.

Example. Let us introduce the concept of table insertion. To do this, first the table rows are inserted, that is, the smallest element of each row of the table is subtracted from the corresponding row element. Then the same operation is performed for the table columns, and the table columns are inserted. A table in which all rows and columns are inserted is called inserted. The sum of the smallest elements of the table rows and columns is denoted by h and is called the insertion coefficient of that table. We solve the problem of minimizing the total travel cost by visiting each of the following 8 countries only once and returning to the starting point (for example, Uzbekistan).

List of countries:

1. Uzbekistan (UZ)
2. Saudi Arabia (SA)
3. South Korea (KR)
4. China (CN)
5. Iran (IR)
6. Russia (RU)
7. United States (US)
8. Turkey (TR)

B/T	UZ	SA	KR	CN	IR	RU	US	TR	Smallest by row
UZ(Tashkent)		450	550	500	300	250	900	250	250
IN(Riot)	450		700	650	400	500	950	300	300
KR(Seoul)	550	700		300	600	100	1000	750	100
CN(Beijing)	500	650	300		550	500	950	700	300
IR(Tehran)	300	400	600	550		350	850	200	200
RU (Moscow)	250	500	600	500	350		800	200	200
US(Nyu-York)	900	950	1000	950	850	800		850	800
TR(Istanbul)	250	300	750	700	200	200	850		200

Table 1.

To get the rows of Table 1, we write the smallest element of the corresponding row on its right side and subtract it from the row elements, resulting in the following table 2.

B/T	UZ	SA	KR	CN	IR	RU	US	TR
UZ		200	300	250	50	0	650	0
SA	150		400	350	100	200	650	0
KR	450	600		200	500	0	900	650
CN	200	350	0		250	200	650	400
IR	100	200	400	350		150	650	0
RU	50	300	400	300	150		600	0
US	100	150	200	150	50	0		50
TR	50	100	550	500	0	0	650	
smallest element by column	50	100	0	150	0	0	600	0

Table 2

In order to bring the columns of the resulting table 2, the smallest element of the corresponding column is written below the table and subtracted from the elements of the column, resulting in the following table 3.

B/T	UZ	SA	KR	CN	IR	RU	US	TR
UZ		100	300	100	50	$0^{(0)}$	50	$0^{(0)}$
SA	100		400	200	100	200	50	$0^{(50)}$
KR	400	500		50	500	$0^{(50)}$	300	650
CN	150	250	$0^{(250)}$		250	200	50	400
IR	50	100	400	200		150	50	$0^{(50)}$
RU	$0^{(0)}$	200	400	150	150		$0^{(50)}$	$0^{(0)}$
US	50	50	200	$0^{(50)}$	50	$0^{(50)}$		50
TR	$\overline{(4,7)}$	$0^{(50)}$	550	350	$0^{(50)}$	$0^{(0)}$	50	

Table 3

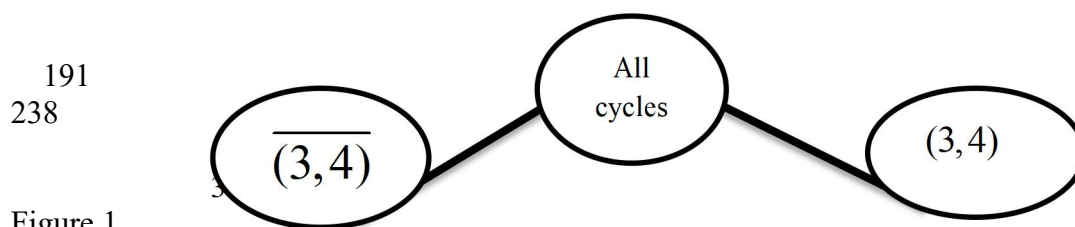


Figure 1.

Table 3 is given, each row and column of which has at least one zero element. The coefficient of occurrence h of the table under consideration is equal to the following number $h_1 = 250 + 300 + 100 + 300 + 200 + 200 + 800 + 200 + 50 + 100 + 150 + 600 = 3250$

$$h_1' = h_1 + 250 = 3500$$

In general, the method of branches and boundaries consists of two important stages:

- 1) branching;
- 2) determination of lower bounds.

During the solution of the problem, both stages are carried out in parallel. To implement these stages, the following steps must be performed sequentially. A) derive the initial table; B) determine the coefficient of the derivative h ; C) determine the level of the zero elements of the derived table; D) perform branching based on these levels; E) determine the lower bounds of the cycles that make up the branching results; F) reduce the size of the table by one; G) avoid the formation of incomplete cycles; H) continue this process until a (2x2) table is formed; I) determine the cycle corresponding to the last branching result; J) compare all the bounds (values); K) if necessary, restore the table corresponding to the lowest boundary result and continue branching.

When using this method, all calculations are carried out using a given table, and the results are displayed on a separate graph. At the end of this process, the perfect (minimum cost) cycle is determined.

The graph consists of interconnected circles, each of which defines a set of cycles with a certain property. The boundary numbers written next to these circles indicate the lower limit of the costs corresponding to the cycles belonging to this circle. The initial part of the graph looks like Figure 1. Here, the first initial circle defines the set containing all cycles and indicates that the cost of an arbitrary cycle cannot be less than the number h . In the example above, $h=3250$, which means that there is no cycle with a cost less than 3250.

The row with the highest rank containing zero i and column j are found, (i, j) If there are multiple high-order zeros, one of them is chosen at random. Here, the circle on the right represents the set of all cycles that include a transition from city i to city j , and it (i, j) is denoted by \circ , while the circle on the left, on the contrary, denotes the set of routes that do not include a transition from city i to city j , and it (i, j) is denoted by \ominus .

The zero element with the highest degree $250 \quad c_{3,4} = 0^{(250)}$, so the branching graph is as shown in Figure 1. The coefficient of least cost for the left-hand path is $h = 3250$ The greatest degree of zero in 250 formed by joining 3500 the number is written. (h_1') To determine the lower limit of the costs corresponding to the circle on the right, row 3 and column 2 of table 3 are deleted (i.e., the size of the table is reduced by one). It should be noted that the ordinal numbers of the cities must be preserved, otherwise there will be confusion. After that, all incomplete cycles are prohibited, for example $i \rightarrow j \rightarrow i(i \rightarrow j)$ (the sign indicates the transition from city to city) is lost, for this c_{ji} element \quad is replaced by the symbol and written, $c_{43} = \quad$).

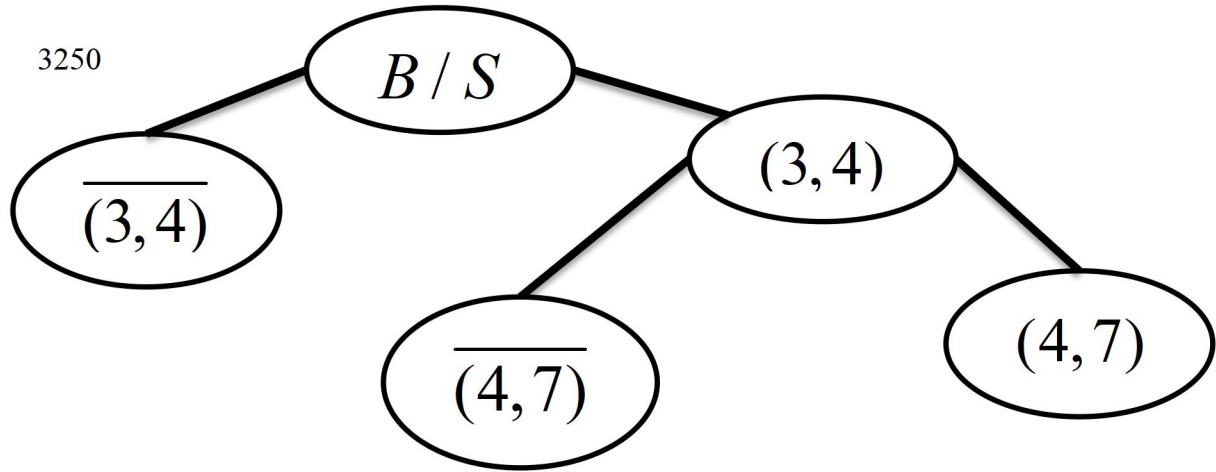
We will continue our work by creating more tables.

B/T	1	2	4	5	6	7	8	
1		100	100	50	0	50	0	0
2	100		200	100	200	50	0	0
3	400	500		500	0	300	650	0
5	50	100	200		150	50	0	0
6	0	200	150	150		0	0	0

7	50	50	0	50	0		50	0
8	0	0	350	0	0	50		0
	0	0	0	0	0	0	0	

Table 4.

$$h_2 = 3250 + 0 = 3250 \quad h_2' = h_2 + 100 = 3350$$



3250

B/T	1	2	4	5	6	7	8
1		100	100	50	0	50	0
2	100		200	100	200	50	0 ⁽⁵⁰⁾
3	400	500		500	0 ⁽⁰⁾	300	0
5	50	100	200		150	50	0 ⁽⁵⁰⁾
6	0 ⁽⁰⁾	200	150	150		0 ⁽⁵⁰⁾	0 ⁽⁰⁾
7	50	50	0 ⁽¹⁰⁰⁾	50	0 ⁽⁰⁾		50
8	0 ⁽⁰⁾	0 ⁽⁵⁰⁾	350	0 ⁽⁵⁰⁾	0 ⁽⁰⁾	50	

3350

Figure 2.

Table 5.

$$c_{63} = 0^{(300)}$$

We can also compile Figure 2 and similar graphs at the end.

B/T	1	2	5	6	7	8	
1		100	50	0 ⁽⁰⁾	50	0 ⁽⁰⁾	0
2	100		100	200	50	0 ⁽⁵⁰⁾	0
3	400	500	500	0 ⁽⁴⁰⁰⁾		0	0
5	50	100		150	50	0 ⁽⁵⁰⁾	0
6	0 ⁽⁰⁾	200	150		0 ⁽⁵⁰⁾	0 ⁽⁰⁾	0
8	0 ⁽⁰⁾	0	0 ⁽³⁰⁾	0 ⁽⁰⁾	50		0
	0	0	0	0	0	0	

Table 6.

$$h_3 = 3250 \quad h_3' = 3250 + 400 = 3650$$

$$c_{63} = 0^{(400)}$$

B/T	1	2	5	7	8	
1		100	50	50	0	0
2	100		100	50	0	0
5	50	100		50	0	0
6	0	200	150		0	0
7	0	0	0	50		0
	0	0	0	50	0	

Table 7

B/T	1	2	5	7	8
1		100	50	$0^{(0)}$	$0^{(0)}$
2	100		100	$0^{(0)}$	$0^{(0)}$
5	50	100		$0^{(0)}$	$0^{(0)}$
6	$0^{(0)}$	200	150		$0^{(0)}$
8	$0^{(0)}$	$0^{(100)}$	$0^{(50)}$	$0^{(0)}$	

Table 8

$$h_4 = 3250 + 50 = 3300 \quad h_4' = 3300 + 100 = 3400$$

$$c_{28} = 0^{(100)}$$

B/T	1	5	7	8	
1		50	0	0	0
2	100	100	0		0
5	50		0	0	0
6	0	150		0	0
	0	50	0	0	

Table 9

B/T	1	5	7	8
1		$0^{(100)}$	$0^{(0)}$	$0^{(0)}$
2	100	50	$0^{(50)}$	
5	50		$0^{(0)}$	$0^{(0)}$
6	$0^{(50)}$	100		$0^{(0)}$

Table 10

$$h_5 = 3300 + 50 = 3350 \quad h_5 = 3300 + 50 = 3350$$

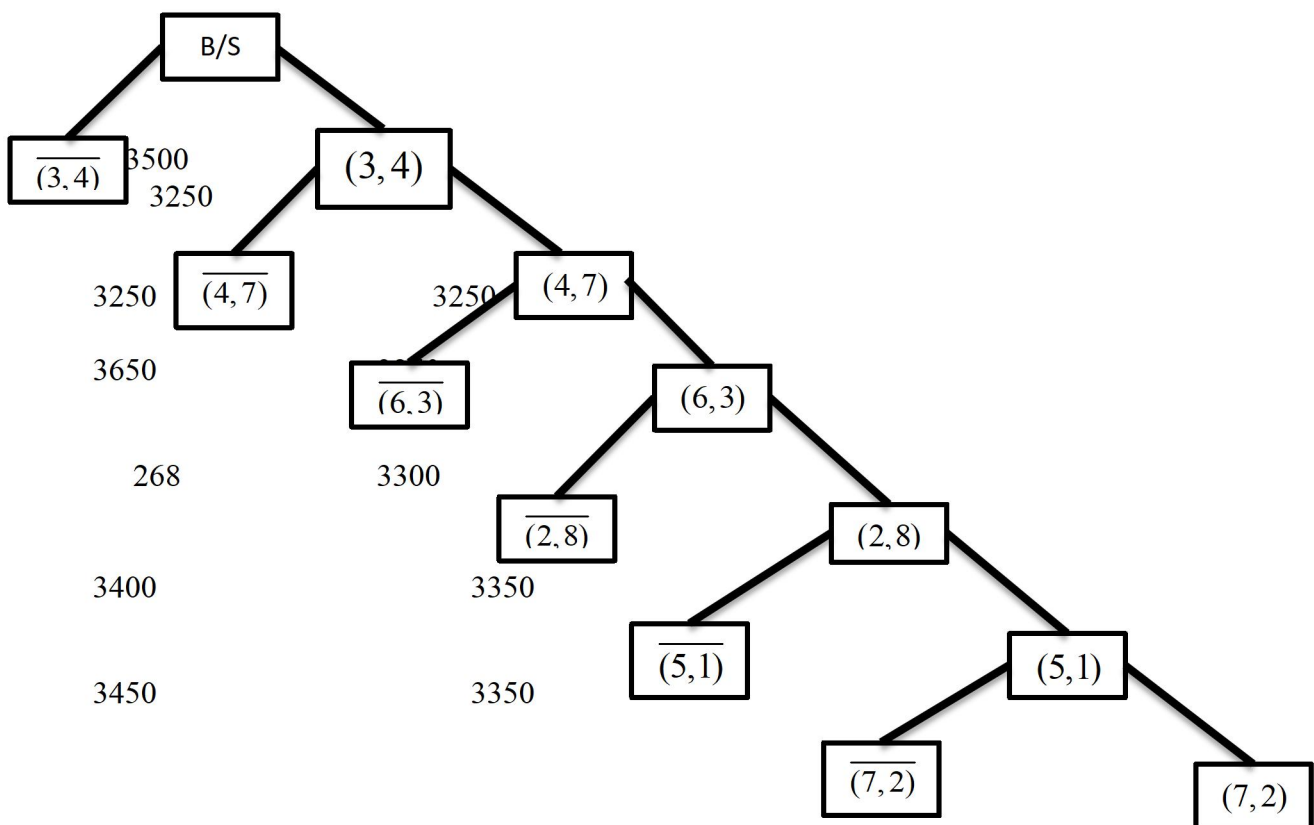
$$c_{51} = 0^{(50)}$$

B/T	1	7	8	
2	100	$0^{(100)}$		0
5		$0^{(0)}$	$0^{(0)}$	0
6	$0^{(100)}$		$0^{(0)}$	0
	0	0	0	

Table 11

$$h_6 = 3350 \quad h_6' = 3450 \quad c_{72} = 0^{(100)}$$

B/T	1	8
5		0
6	0	



Shortest path (optimal route):

$1 \rightarrow 6 \rightarrow 3 \rightarrow 4 \rightarrow 7 \rightarrow 2 \rightarrow 8 \rightarrow 5 \rightarrow 1$

Minimum distance: 3850 unity.

Conclusion. The traveling salesman problem is a classic mathematical problem used in modeling and optimizing many real-world logistics and planning problems. Due to its computational complexity, heuristic or approximate algorithms are often used in practical

solutions. In this work, the theoretical and practical aspects of the problem were studied and an effective result was achieved by determining the optimal path.

References:

1. Papadimitriou, C. H., & Steiglitz, K. (1998). *Combinatorial Optimization: Algorithms and Complexity*. Dover Publications.
2. Applegate, D., Bixby, R., Chvátal, V., & Cook, W. (2006). *The Traveling Salesman Problem: A Computational Study*. Princeton University Press.
3. Lawler, E. L., Lenstra, J. K., Rinnooy Kan, A. H. G., & Shmoys, D. B. (1985). *The Traveling Salesman Problem: A Guided Tour of Combinatorial Optimization*. Wiley.
4. Gutin, G., & Punnen, A. P. (2002). *The Traveling Salesman Problem and Its Variations*. Springer.
5. Reinelt, G. (1994). *The Traveling Salesman: Computational Solutions for TSP Applications*. Springer.