JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS
GERMAN INTERNATIONAL JOURNALS COMPANY
ISSN: 2751-4390
IMPACT FACTOR (RESEARCH BIB) : 9,08. Academic reserach index

MATHEMATICAL MODEL OF THE TRANSPORTATION PROBLEM AND FINDING THE OPTIMAL SOLUTION

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Annotation: This article is dedicated to the "Transportation Problem"**,** one of the key areas of operations research and optimal control. It explores the theoretical foundations, mathematical models, and practical applications of this problem. The transportation problem is considered an optimal resource allocation problem and is widely applied in fields such as economics, logistics, and supply chain management. The article provides a detailed explanation of the main solution methods for the transportation problem, including the “Northwest Corner Method”, “Minimum Cost Method”, and “Potential Method”**.** The principles for determining the optimal solution using these methods are analyzed. Additionally, practical examples of the transportation problem’s application in real-world economics and logistics are presented, illustrating the efficiency and applicability of these methods. The research findings demonstrate that understanding and solving the transportation problem can help optimize logistics systems, minimize transportation costs, and improve resource utilization efficiency. This article serves as an important theoretical and practical guide for researchers, economists, and logistics specialists working with transportation problems.
Keywords: transportation problem, operations research, optimal control, optimal allocation, logistics, efficient resource allocation, Northwest Corner Method, Minimum Cost Method, Potential Method, mathematical modeling, freight transportation problem, optimal solution, transportation cost minimization.

The general formulation of the transportation problem is as follows: $A_1, A_2, A_3, \dots A_m$ Am are supply points dealing with the same type of product, where A_i – represents the amount of product at point a_i and the quantity is denoted as ai units. The task is to distribute these products to the consumption points$B_1, B_2, B_3, \dots B_n$. At each consumption point B_j – the required amount of product to be delivered is b_j units.

Let c_{ij} be the cost in soums to transport one unit of product from supply point A_i to consumption point B_j. The task is to distribute the products from the supply points to the consumers with the minimum total cost. To solve this problem, the amount of product to be transported from A_i to B_j, denoted as x_{ij}, is determined. This leads to the construction of the mathematical model for the problem,

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} \to \min \qquad (1)$$

$$\sum_{j=1}^{n} x_{ij} = a_i , \; i = 1,2,...,m, \qquad (2)$$

$$\sum_{i=1}^{m} x_{ij} = b_j , \; j = 1,2,...,n, \qquad (3)$$

$$x_{ij} \geq 0, \; i = 1,2,...,m, \;\; j = 1,2,...,n, \qquad (4)$$

Here, (2) represents the constraint on the amount of product to be taken by each supplier, and (3) represents the constraint on the amount of product to be delivered to each consumer. (1) is called the objective function, which determines the total transportation cost. Therefore, (1) indicates that we need to minimize this total cost.

(2), (3), and (4) represent the set of constraints that define the feasible solutions for the transportation problem. The set of plans corresponding to this feasible solution is called the plan set. Each xij vector in the plan set is referred to as a transportation plan for the corresponding transportation problem.

Thus, the formulation of the transportation problem is as follows: a plan must be selected from the plan set such that it minimizes the value of the objective function (1).

Definition 1: If equality $\sum_{i=1}^{m} a_i = \sum_{j=1}^{n} b_j$ is satisfied, the corresponding transportation problem is called a closed transportation problem. Otherwise, it is called an open transportation problem.

Theorem 1: Any closed transportation problem has a solution. If the transportation problem is open, it can be transformed into a closed transportation problem and solved. There are several methods for solving transportation problems, and we can gain detailed information about these methods by solving the problem presented below.

Problem: A large logistics company provides product delivery services in Uzbekistan. The company has three warehouses located in the cities of Tashkent, Samarkand, and Bukhara. Products need to be delivered from these warehouses to three cities – Fergana, Karshi, and Nukus. The objective is to minimize the total transportation costs.

The shipping capabilities of the warehouses (in units):

Tashkent: 200 units

Samarkand: 300 units

Bukhara: 500 units

Demand quantities (in units):

- Fergana: 250 units
- Karshi: 350 units
- Nukus: 400 units

Solution of the problem:

Consumer / **Supplier**		**Andijan**	**Bukhara**	**Karshi**	**Nukus**	**Available product quantites**
		B_1	**B_2**	**B_3**	**B_4**	
Tashkent	**A_1**	12$	18$	25$	20$	200
Samarkand	**A_2**	10$	14$	15$	22$	150
Fergana	**A_3**	8$	16$	24$	30$	250
Demands		180	120	170	130	600

North-West Corner Method: This method starts by satisfying the demand of consumer b_1 with the available product from supplier A_1. If the demand is satisfied (for this to happen a_1 b_1 the quantity should be sufficient), then the remaining product from $A_1$is used to satisfy the demand of consumer B_2, and so on. If supplier A_1cannot fully satisfy the demand of consumer B_1, then the product from supplier $A_2$is used, and with its help, the demand of B_1 is either fully or partially satisfied. Since a closed transportation problem is being considered, this process continues until all the available products from the suppliers are fully distributed to the consumers. In each step of this process, either the product of the corresponding supplier is fully distributed, or the demand of the corresponding consumer is fully satisfied.

This method starts by satisfying the demand of consumer b_1 with the available product from supplier A_1. If the demand is satisfied (for this to happen, a_1 b_1 must hold), then the remaining product from $A_1$is used to satisfy the demand of consumer B_2, and so on. If supplier A_1 cannot fully satisfy the demand of consumer B_1, then the product from supplier A_2 is used, and with its help, the demand of B_1 is either fully or partially satisfied. Since a closed transportation problem is being considered, this process continues until all the available products from the suppliers are fully distributed to the consumers. In each step of this process, either the product of the corresponding supplier is fully distributed, or the demand of the corresponding consumer is fully satisfied.

Consummer / Supplier		Andijan	Bukhara	Karshi	Nukus	Available product quantites
		B_1	B_2	B_3	B_4	
Tashkent	A_1	12 180	18 20	25 0	20 0	200
Samarkand	A_2	10 0	14 100	15 50	22 0	150
Fergana	A_3	8 0	16 0	24 120	30 130	250
Demands		180	120	170	130	600

If both situations occur simultaneously — that is, if the available products of several suppliers are fully distributed while the corresponding consumers' demands are also fully satisfied — then a zero is written in the corresponding right or lower cell. By assigning such zeros in some cells, a special initial feasible solution is constructed. This method will be explained through the following example: Let A_1, A_2, A_3 be the suppliers with available product quantities of $a_1=200$, $a_2=150$, $a_3=250$ units, respectively. The consumers B_1, B_2, B_3, B_4 have product demands of $b_1=180$, $b_2=120$, $b_3=170$, $b_4=130$ units, respectively. The cost cij of transporting one unit of product from supplier A_i to consumer B_j is given by the following values: c_{11}=12$, c_{12}=18$, c_{13}=25$, c_{14}=20$, c_{21}=10$, c_{22}=14$, c_{23}=15$, c_{24}=22$, c_{31}=8$, c_{32}=16$, c_{33}=24$, c_{34}=30$

This problem was placed into our first table. Now, in the second table, we will explain how to work using the "North-West Corner Method."
The available product quantity at supplier A_1 is 200 units. We begin by satisfying the demand of the consumer with the smallest index, B_1, using these products.
Out of the 200 units, 180 units are allocated to B_1, and the remaining 20 units are given to B_2. Then, the unmet 100 units of demand for B_2 are fulfilled using the product from supplier A2. The remaining quantity of supplier A_2, which is $150 - 100 = 50$ units, is allocated to satisfy part of the demand of B_3.
The remaining demand of B_3, $170 - 50 = 120$ units, is satisfied by supplier A_3.
The remaining product from A_3, $250 - 120 = 130$ units, is sufficient to meet the demand of B_4.

These values are recorded in the second table. From the table, it is clear that, based on this process, we have obtained an initial basic feasible solution: $x_{11}=180$, $x_{12}=20$, $x_{13}=0$, $x_{14}=0$, $x_{21}=0$, $x_{22}=100$, $x_{23}=50$, $x_{24}=0$, $x_{31}=0$, $x_{32}=0$, $x_{33}=120$, $x_{34}=130$. To determine the total transportation cost corresponding to this initial solution, we use the following formula:

$$\sum_{i=1}^{m}\sum_{j=1}^{n} c_{ij}x_{ij} = c_{11}\bullet x_{11} + c_{12}\bullet x_{12} + \ldots + c_{33}\bullet x_{33} + c_{34}\bullet x_{34}$$

If we denote this expression by **Z**, then the total transportation cost for our transportation problem will be equal to the following value:

Z=180·12+20·18+0·25+0·20+0·10+100·14+50·15+0·22+0·8+0·16+120·24+130·30=2160+360+0 +0+0+1400+750+0+0+0+2880+3900=11450$

Thus, for the given example, the total transportation cost corresponding to the obtained initial solution amounts to **$11,450**.

Conclusion

This article discusses the transportation problem, where the transportation costs were calculated using the North-West Corner Method.
The main focus was not on delivering products with minimal cost, but rather on supplying consumers sequentially according to their demands, without prioritizing cost minimization. In this approach, suppliers deliver products to consumers step-by-step, satisfying the demands in order.

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