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# DECISION MAKING UNDER UNCERTAINTY

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**Annotation:** In this article, we will explore various criteria encountered in decision-making problems involving nature. These include the expected value (mathematical expectation) criterion, the Laplace criterion, Wald's minimax (maximin) criterion, the Savage criterion, the Hurwicz criterion, and the Hodges-Lehmann criterion.
**Keywords:** Game against nature, expected monetary value, Laplace criterion, Wald's minimax (maximin) criterion, Savage criterion, Hurwicz criterion, Hodges-Lehmann criterion.

**Annotatsiya:** Biz bu maqolada tabiat bilan o'yin masalasida uchraydigan turli kriteriyalar bilan tanishib chiqamiz. Bular yutuqning matematik kutilmasi kriteriyasi, Laplas kriteriyasi, Valdning minimaks (maksimin) kriteriyasi, Sevidj kriteriyasi, Gurvits kriteriyasi va Xodja-Leman kriteriyasi.
**Kalit so'zlar:** Tabiat bilan o'yin, yutuqning matematik kutilmasi, Laplas kriteriyasi, Valdning minimaks (maksimin) kriteriyasi, Sevidj kriteriyasi, Gurvits kriteriyasi, Xodja-Leman kriteriyasi.

**Аннотация:** В данной статье мы ознакомимся с различными критериями, возникающими при решении задач взаимодействия с природой. Это критерий математического ожидания выигрыша, критерий Лапласа, минимаксный (максиминный) критерий Вальда, критерий Сэвиджа, критерий Гурвица и критерий Ходжа-Лемана.
**Ключевые слова:** Игра с природой, математическое ожидание выигрыша, критерий Лапласа, критерий Вальда минимакс (максимин), критерий Савиджа, критерий Гурвица, критерий Ходжа-Лемана.

## Introduction

Decision Making under Risk - A Game with Nature

The states are known and defined by $\theta_1, \theta_2, ..., \theta_n$. Let the decisions (solutions) we make be $\alpha_1, \alpha_2, ..., \alpha_m$. Suppose that when we make the $\alpha_i$decision, nature brings about the $\theta_j$ state. In this case, the benefit (profit, income, gain) we receive will be equal to $w_{ij}$. This can be expressed in the following table:

| | $\theta_1$ | $\theta_2$ | ... | $\theta_n$ |
|---|---|---|---|---|
| $\alpha_1$ | $w_{11}$ | $w_{12}$ | ... | $w_{1n}$ |
| $\alpha_2$ | $w_{21}$ | $w_{22}$ | ... | $w_{2n}$ |
| ⋮ | ⋮ | ⋮ | ⋮ | ⋮ |

| | | | | |
|---|---|---|---|---|
| $\alpha_m$ | $w_{m1}$ | $w_{m2}$ | $\cdots$ | $w_{mn}$ |

**Objective:** The goal in a game with nature is to choose one of the possible solutions $\theta_1, \theta_2, \ldots, \theta_n$, without knowing which state $\alpha_1, \alpha_2, \ldots, \alpha_m$ nature will bring about, in such a way that the resulting gain is maximized. To achieve this objective, several methods—mentioned above—have been proposed. We will now examine each of these methods one by one.

1. Hurwicz Method: This method depends on a $0 \le \beta \le 1$ parameter, which indicates the degree of "optimism" of the decision-maker. First, based on the value of $\beta$, the differences $i = 1,2, \ldots, m$ are calculated for all values of

$$w_i = \beta \max_{j=\overline{1n}} w_{ij} + (1-\beta) \min_{\overline{jn}} w_{ij}.$$

Then, the value of $w_i$ that maximizes $i$ is determined, and the corresponding $\alpha_i$ is selected.

2. Method of Maximizing the Expected Value: In this method, it is assumed that $\theta_1, \theta_2, \ldots, \theta_n$ the probabilities of the possible states occurring are known, and let them be $p_1, p_2, \ldots, p_n$ accordingly. In that case, by choosing decision $\alpha_i$, one obtains an average gain of $w_i = \sum_{j=1}^{n} p_j w_{ij}$. The maximum among these $w_k$ values determines the decision $\alpha_k$ that should be selected.

3. Laplace Method: This method is a special case of the method of maximizing the expected value $p_1 = p_2 = \ldots = p_n = 1/n$ in which $\theta_1, \theta_2, \ldots, \theta_n$ the probabilities of the possible states are assumed to be equal.

4. Minimax and Maximin Methods: $W$ the decision determined by the minimum of the row-wise maximum values in the payoff table is called the minimax decision. $\alpha_k$ the decision determined by the maximum of the row-wise minimum values in the payoff table is called the maximin decision.

5. Savage Method: In the Savage method, a regret table R is constructed based on the following rule: $r_{ij} = \max_{l=\overline{1m}} w_{lj} - w_{ij}$. The maximin method is then applied to this table to determine the optimal decision $\alpha_k$.

6. Hodges–Lehmann Method: In this method, a parameter $0 \le \gamma \le 1$ is involved, and its value determines the confidence level of the probabilitie $p_1, p_2, \ldots, p_n$ that represent the likelihood of the different states $\theta_1, \theta_2, \ldots, \theta_n$ occurring. The corresponding decision $\alpha_k$ is determined by finding the maximum of the values based on $w_i = \gamma \sum_{j=1}^{n} p_j w_{ij} + (1-\gamma) \min_{j=\overline{1n}} w_{ij}$.

**Sample Problem:** Let's consider the example given in the table below using the six different methods discussed above.

Given Probabilities: $p_1 = \frac{1}{3}$, $p_2 = \frac{1}{6}$, $p_3 = \frac{1}{4}$, $p_4 = \frac{1}{4}$

Coefficients: $\beta = \frac{2}{3}$, $\gamma = \frac{2}{3}$

| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
|---|---|---|---|---|
| $\alpha_1$ | 4 | 0 | 5 | 2 |
| $\alpha_2$ | 2 | 3 | 1 | 4 |
| $\alpha_3$ | 3 | 2 | 6 | 1 |

**1. Hurwics Method**

$w^*_1=\max_{j=\overline{1n}} w_{1j}=\max_{j=\overline{1n}}(4,0,5,2)=5$

$w^*_2=\max_{j=\overline{1n}} w_{2j}=\max_{j=\overline{1n}}(2,3,1,4)=4$

$w^*_3=\max_{j=\overline{1n}} w_{3j}=\max_{j=\overline{1n}}(3,2,6,1)=6$

$w_{1*}=\min_{j=\overline{1n}} w_{1j}=\min_{j=\overline{1n}}(4,0,5,2)=0$

$w_{2*}=\min_{j=\overline{1n}} w_{2j}=\min_{j=\overline{1n}}(2,3,1,4)=1$

$w_{3*}=\min_{j=\overline{1n}} w_{3j}=\min_{j=\overline{1n}}(3,2,6,1)=1$

the following formula of the Hurwicz criterion

$$w_i = \beta \max_{j=\overline{1n}} w_{ij} + (1-\beta)\min_{\overline{jn}} w_{ij}$$

according to ,

$w_1=\beta w^*_1+(1-\beta)w_{1*}=\frac{2}{3}*5+\frac{1}{3}*0=\frac{10}{3}$

$w_2=\beta w^*_2+(1-\beta)w_{2*}=\frac{2}{3}*4+\frac{1}{3}*1=3$

$w_3=\beta w^*_3+(1-\beta)w_{3*}=\frac{2}{3}*6+\frac{1}{3}*1=\frac{13}{3}$

*These can also be written in general form as follows:*

$\max\left\{\frac{2}{3}(5,4,6)+\frac{1}{3}(0,1,1)\right\} = \max\left(\frac{10}{3},3,\frac{13}{3}\right) = \frac{13}{3}$,

Thus, it follows that the decision-maker should choose strategy $\alpha_3$

**2. Method of Maximizing the Expected Value:**

*In this method, the decision is determined by finding the maximum* $w_i$ *of the formula for solution* $\alpha_k$

$$w_i = \sum_{j=1}^{n} p_j w_{ij}$$

$w_1=p_1w_{11}+p_2w_{12}+p_3w_{13}+p_4w_{14}=\frac{1}{3}*4+\frac{1}{6}*0+\frac{1}{4}*5+\frac{1}{4}*2=\frac{37}{12}$

$w_2=p_1w_{21}+p_2w_{22}+p_3w_{23}+p_4w_{24}=\frac{1}{3}*2+\frac{1}{6}*3+\frac{1}{4}*1+\frac{1}{4}*4=\frac{29}{12}$

$w_3=p_1w_{31}+p_2w_{32}+p_3w_{33}+p_4w_{34}=\frac{1}{3}*3+\frac{1}{6}*2+\frac{1}{4}*6+\frac{1}{4}*1=\frac{37}{12}$

$\max_{i=1,2,3} w_i=\max_{i=1,2,3}(\frac{37}{12}, \frac{29}{12}, \frac{37}{12})=\frac{37}{12}$

**3. Laplace Method:**

In this method, the decision-maker's $\alpha_i$ strategy provides an average gain of $\frac{1}{n}\sum_{j=1}^{n} w_{ij}$ o. Therefore, they will choose the $\alpha_k$ strategy that maximizes this average gain,

$$\max_{\alpha_i} \frac{1}{n}\sum_{j=1}^{n} w_{ij} = \frac{1}{n}\sum_{j=1}^{n} w_{kj}$$

In our given example, based on the Laplace method,

$$\max_{\alpha_i} \frac{1}{4}\sum_{j=1}^{3} w_{ij} = \max\frac{1}{4}(11,10,12) = \max(\frac{11}{4},\frac{5}{2},3) = 3$$

Therefore, according to the Laplace criterion, the decision-maker should apply strategy $\alpha_3$.

**4. Maximin (Minimax) Method:**

In this criterion, if the decision-maker applies strategy $\alpha_i$ and an unfavorable state occurs due to nature, their gain will be

$w_i = \min_j w_{ij}$.

Therefore, they will try to apply such a strategy $\alpha_{\text{i}}$ that the maximum of the minimum gains is determined,

$$w_* = \max_i w_i = \max_i \min_j w_{ij}$$

the decision that ensures the maximum value of $\alpha_k$ is considered the optimal strategy of the decision-maker.

Based on the above, by applying strategy $\alpha_k$, the decision-maker is guaranteed to achieve at least $w_* -$ in guaranteed gain.

Now, for each given $i = 1,2,3$ in the example, let's determine$\min_j w_{ij}$ :

$\min_j w_{1j} = \min(4,0,5,2) = 0,\ \min_j w_{2j} = \min(2,3,1,4) = 1,\ \min_j w_{3j} = \min(3,2,6,1) = 1.$ This

implies that,

$$w_k = \max_i \min_j w_{ij} = \max\left(\min_j w_{1j}, \min_j w_{2j}, \min_j w_{3j}\right) = \max(0,1,1) = 1$$

Therefore, the decision-maker's optimal maximin strategy is $\alpha_2$ and $\alpha_3$, and their guaranteed gain is equal to 1.

Note: If the elements of the table $W = (w_{ij})$ represent the decision-maker's cost (loss, defeat), then, using the above reasoning, the guaranteed cost will be equal to the

$$\min_j \max_i w_{ij} = \min\left(\max_i w_{i1}, \max_i w_{i2}, \max_i w_{i3}\right) = \min(5,4,6) = 4$$

In this case, the strategy to be chosen will be $\alpha_2$, but

$\min_j \max_i w_{ij} \neq \max_i \min_j w_{ij}$.

**5. Savage Method:**

In the Savage method, a table called "regret" is constructed based on the following rule: $r_{ij} = \max_{l=\overline{1m}} w_{lj} - w_{ij}$. The maximin method is applied to the resulting table, and the decision $\alpha_k$ is determined.

It is known that the number $\max_i \min_j w_{ij}$ represents the guaranteed gain of the decision-maker.

$\max_{i=1,2,3} w_{i1} = \max_{i=1,2,3}(4,2,3) = 4$ ,  $\max_{i=1,2,3} w_{i2} = \max_{i=1,2,3}(0,3,2) = 3$

$\max_{i=1,2,3} w_{i3} = \max_{i=1,2,3}(5,1,6) = 6$ ,  $\max_{i=1,2,3} w_{i4} = \max_{i=1,2,3}(2,4,1) = 4$

The elements of the table $R = (r_{ij})$ are obtained by subtracting each column element of the table $W = (w_{ij})$ from the largest element in that column (as previously derived).

$r_{11} = \max_{i=1,2,3} w_{i1} - w_{11} = 4-4=0$ ,  $r_{21} = \max_{i=1,2,3} w_{i1} - w_{21} = 4-2=2$,

$r_{31} = \max_{i=1,2,3} w_{i1} - w_{31} = 4-3=1$

$r_{12} = \max_{i=1,2,3} w_{i2} - w_{12} = 3-0=3$ ,  $r_{22} = \max_{i=1,2,3} w_{i2} - w_{22} = 3-3=0$

$r_{32} = \max_{i=1,2,3} w_{i2} - w_{32} = 3-2=1$

$r_{13} = \max_{i=1,2,3} w_{i3} - w_{13} = 6-5=1$ ,  $r_{23} = \max_{i=1,2,3} w_{i3} - w_{23} = 6-1=5$

$r_{33} = \max_{i=1,2,3} w_{i3} - w_{33} = 6-6=0$

$r_{14} = \max_{i=1,2,3} w_{i4} - w_{14} = 4-2=2$ ,  $r_{24} = \max_{i=1,2,3} w_{i4} - w_{24} = 4-4=0$

$r_{34} = \max_{i=1,2,3} w_{i4} - w_{34} = 4-1=3$

The general form of the "regret" table R will be as follows:

| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
|---|---|---|---|---|
| $\alpha_1$ | $r_{11}$ | $r_{12}$ | $r_{13}$ | $r_{14}$ |
| $\alpha_2$ | $r_{21}$ | $r_{22}$ | $r_{23}$ | $r_{24}$ |

| $\alpha_3$ | r31 | r32 | r33 | r3 |
|---|---|---|---|---|

If the obtained values are placed in the table accordingly, it will look as follows:

| | $\theta_1$ | $\theta_2$ | $\theta_3$ | $\theta_4$ |
|---|---|---|---|---|
| $\alpha_1$ | 0 | 3 | 1 | 2 |
| $\alpha_2$ | 2 | 0 | 5 | 0 |
| $\alpha_3$ | 1 | 1 | 0 | 3 |

$r_{1*}$=min(0,3,1,2)=0 , $r_{2*}$=min(2,0,5,0)=0 , $r_{3*}$=min(2,1,0,3)=0

It is known that the number $\max_i \min_j r_{ij}$ represents the guaranteed gain of the decision-maker.

$$\max_i \min_j r_{ij} = \max(0,0,0) = 0$$

Therefore, in the Savage method, the decision-maker's strategy is $\alpha_1, \alpha_2, \alpha_3$.

**6. Hodges–Lehmann Method:**

The decision corresponding to this method is determined by finding the maximum of the values $w_i = \gamma\sum_{j=1}^{n} p_j w_{ij} + (1-\gamma)\min_{j=\overline{1n}} w_{ij}$ of the solution $\alpha_k$

It is given that it is equal to $\gamma=\frac{2}{3}$.

$w_1 = \gamma w_1 + (1-\gamma) w_{1*} = \frac{2}{3} * \frac{37}{12} + \frac{1}{3} * 0 = \frac{37}{18}$

$w_2 = \gamma w_2 + (1-\gamma) w_{2*} = \frac{2}{3} * \frac{29}{12} + \frac{1}{3} * 1 = \frac{35}{18}$

$w_3 = \gamma w_3 + (1-\gamma) w_{3*} = \frac{2}{3} * \frac{37}{12} + \frac{1}{3} * 1 = \frac{43}{18}$

$\max_{i=1,2,3} w_i = \max(\frac{37}{18}, \frac{35}{18}, \frac{43}{18}) = \frac{43}{18}$

Therefore, according to the Hodges-Lehmann criterion, the decision-maker should apply strategy $\alpha_3$.

**Conclusion**

The problem under consideration has been framed as a two-player game, where the second player is considered the "unaware" opponent. This type of game is called a game with nature. The most important aspect of such a game is that nature, by bringing about various states, is not interested in which of these states occurs. The main issue in the game with nature is to define a goal-oriented criterion and find the optimal solution relative to it.

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