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UNIQUENESS OF THE SOLUTION OF A BOUNDARY VALUE PROBLEM FOR A NONCLASSICAL PARABOLIC-TYPE EQUATION

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Abstract: This paper investigates the uniqueness of the solution to a boundary value problem for a nonclassical parabolic-type differential equation. The problem is considered in a general form, and through appropriate substitutions the equation is simplified and formulated in a given domain with specified boundary conditions. The uniqueness of the solution is proved using the energy integral method, and it is shown that the corresponding homogeneous problem admits only the trivial solution. The obtained results are of significant importance in the theory of equations of mathematical physics and can be applied to the study of nonclassical boundary value problems.

Keywords: parabolic equation, nonclassical equation, boundary value problem, uniqueness theorem, energy integral method, homogeneous problem, trivial solution, equations of mathematical physics.

Problem statement

We consider the following equation.

$$\frac{\partial^3 u}{\partial x^3}-\frac{\partial^2 u}{\partial y^2}+\sum_{i=0}^{2} a_i(x,y)\frac{\partial^i u}{\partial x^i}+b(x,y)\frac{\partial u}{\partial x}=f(x,y) \qquad (2.1.1)$$

If in equation (2.1.1) $b(x,y)\in C^{3,1}(\overline{D})$ is satisfied

$$u(x,y)=\vartheta(x,y)\exp\left(\frac{1}{2}\int_0^y b(x,t)\,dt\right)$$

without loss of generality, by means of an appropriate substitution

$$\frac{\partial^3 u}{\partial x^3}-\frac{\partial^2 u}{\partial y^2}+\sum_{i=0}^{2} a_i(x,y)\frac{\partial^i u}{\partial x^i}=f(x,y) \qquad (2.1.2)$$

For equation (2.1.2), we consider the following problem in the domain $D=(0<x<1,\ 0<y<1)$
In the domain D, find a boundary value solution $u(x,y)\epsilon C^{2,1}(\overline{D})\cap C^{3,2}(D)$ of equation (2.1.2) such that it satisfies the following boundary conditions [22]:

$$\alpha_0(x)u(x,0)+\alpha_1(x)u(x,1)=\varphi_0(x),\ 0\le x\le 1, \quad (2.1.3)$$
$$\beta_0(x)u(x,1)+\beta_1(x)u_y(x,1)=\varphi_1(x), \quad 0\le x\le 1, \quad (2.1.4)$$
$$\gamma_0(y)u(0,y)+\gamma_1(y)u_x(1,y)=\psi_0(y), \quad 0\le y\le 1, \quad (2.1.5)$$
$$u_x(0,y)=\psi_1(y),\ 0\le y\le 1, \quad (2.1.6)$$
$$u(1,y)=\psi_2(y),\quad 0\le y\le 1, \quad (2.1.7)$$

Here $a_i(x,y), b(x,y), f(x,y), \alpha(x), \beta_i(y), \varphi_i(x_1)$, $(i=0,1)$, $\psi_j(x)$ $(j=0,1,2)$ and $\psi_j(x)$ $(j=0,1,2)$ are given functions continuous in their respective arguments. Moreover, $\alpha_0^2+\alpha_1^2\neq 0$, $\beta_0^2+\beta_1^2\neq 0$, $\gamma_0^2+\gamma_1^2\neq 0$.[

Uniqueness theorem

Suppose that for equation (2.1.1) the functions $a_i(x,y)\in C^{i,0}(\overline{D})$, $a_2(x,y)\le 0$, $a_0-\frac{1}{2}a_{1x}+\frac{1}{2}a_{2xx}\equiv C(x,y)\ge 0$

and that one of the following conditions is satisfied:

1) If $\alpha_0\beta_0\gamma_0\delta_0\neq 0$ and ,

$$\alpha_0\alpha_1\ge 0,\ \beta_0\beta_1\le 0,\ \gamma_0\gamma_1\le 0,\ \gamma_0\gamma_2\ge 0, \delta_0\delta_1\le 0,\ (-1)^i\big(a_1(i,y)-a_{2x}(i,y)\big)\ge 0\quad i=0,1,\ -1\le\frac{\gamma_1}{\gamma_0}+\frac{\gamma_2}{\gamma_0}a_2(0,y);$$

or

2) If $\alpha_1\beta_1\gamma_2\delta_1\neq 0$ and , $\alpha_0\alpha_1\le 0$, $\beta_0\beta_1\ge 0$, $\delta_0\delta_1\le 0$, $\gamma_0\gamma_2\ge 0$, $-1\le\frac{\gamma_1}{\gamma_2}-a_2(0,y)\le 0$, $(-1)^i[a_1(i,y)-a_{2x}(i,y)+i]\ge 0\quad i=0,1.$

Then the solution of problem **A** is unique. Above, we consider only the second case; however, by imposing different conditions on the given functions, various types of problems can be formulated.[1-9].

PROOF OF THE THEOREM

We present the proof of the theorem for the first case. The second case and the remaining cases can be proved in the same manner. The proof is carried out using the energy integral method. For the given problem, we consider the corresponding homogeneous problem and show that it admits only the trivial solution. The homogeneous problem is of the following form: [10-12].

$$L(u)=0 \quad (2.2.1)$$
$$\alpha_0(x)u(x,0)+\alpha_1u_y(x,0)=0,\ \ 0\le x\le 1 \quad (2.2.2)$$
$$\beta_0(x)u(x,1)+\beta_1(x)u_y(x,1)=0,\ \ 0\le x\le 1 \quad (2.2.3)$$
$$\gamma_0(y)u(0,y)+\gamma_1(y)u_y(0,y)+\gamma_2(y)u_{xx}(0,y)=0, \quad (2.2.4)$$
$$\delta_0(y)u(1,y)+\delta_1(y)u_{xx}(1,y)=0, \quad (2.2.5)$$
$$u_x(1,y)=0\ \ 0\le y\le 1, \quad (2.2.6)$$

We show that its solution is $u(x,y)\equiv 0$ We consider the identity $uL(u)=0$ and integrate it over the domain D, obtaining the following results.

$$\iint_{(D)} u\left(\frac{\delta^3 u}{\delta x^3}-\frac{\delta^2 u}{\delta y^2}+a_2(x,y)\frac{\delta^2 u}{\delta x^2}+a_1(x,y)\left(\frac{\delta u}{\delta x}\right)+a_0 u(x,y)\right)dxdy=0$$

By performing the appropriate substitutions in this identity, we obtain the following:

$$\iint_{(D)}\left(u_y^2-a_2 u_x^2+Cu^2\right)dxdy-\int_0^1\left(uu_y\right)_{y=1}dx+\int_0^1\left(uu_y\right)_{y=0}dx+\int_0^1\left[uu_{xx}+a_2 uu_x-\frac{1}{2}u_x^2+\frac{1}{2}(a_1-a_{2x})u^2\right]_{x=1}$$

Using the homogeneous boundary conditions (2.2.2)–(2.2.5) and the inequality

$$kab\geq\frac{k}{2}(a^2+b^2),\ k<0$$

we obtain the following result

$$\iint_D\left(u_y^2-a_2 u_x^2+Cu^2\right)dxdy+\int_0^1\left[\frac{\beta_0(x)}{\beta_1(x)}u^2(x,1)-\frac{\alpha_0(x)}{\alpha_1(x)}u^2(x,0)\right]dx+\int_0^1\left\{\left[\frac{1}{2}a_1(1,y)-\frac{1}{2}a_{2x}(1,y)-\frac{\delta_0(y)}{\delta_1(y)}\right]u^2\right.$$

According to the conditions of the uniqueness theorem, all terms on the left-hand side of inequality (2.2.7) are nonnegative and vanish only when $u(x,y)\equiv 0$ That is,
$U_1(x,y)\equiv\ U_2(x,y)$
This proves the uniqueness of the solution of the problem. [13-18].

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