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DYNAMIC PROPERTIES OF A TOROIDAL SHELL WITH FLOWING INTERNAL FLUID

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Abstract

This paper investigates the dynamics of curved segments of large-diameter thin-walled pipelines modeled as a portion of a toroidal shell conveying an ideal incompressible fluid. Based on the Kirchhoff-Love hypothesis and the relations of the semi-membrane (semi-momentless) shell theory, the equations of motion are derived in toroidal curvilinear coordinates taking into account inertia forces and internal pressure. The pressure acting on the pipe wall is represented as the sum of a constant hydrostatic component and a hydrodynamic component of the flow, expressed via Legendre functions. The solution is obtained by expanding the displacement field in beam-type (fundamental) functions that satisfy the boundary conditions and the cyclicity requirement, reducing the problem to an eigenvalue problem for a matrix whose eigenvalues correspond to the squares of the natural frequencies. The influence of flow velocity, segment curvature, and the shell's relative thickness on the natural frequencies of bending vibrations is analyzed. It is shown that within the range of practical flow velocities the effect of velocity on the natural frequencies is small, whereas increasing curvature and relative thickness leads to higher natural frequencies; for certain velocity values, frequency minima may occur, associated with increased forces and deformations.

Keywords

toroidal shell, thin-walled pipe, curved section, flowing fluid, hydrostatic pressure, hydrodynamic pressure, hydraulic elasticity, natural vibrations, bending vibrations, natural frequencies, toroidal coordinates, Kirchhoff-Love hypothesis, semimoment theory, eigenvalue problem, boundary conditions.

Introduction: Nearly all modern engineering structures and devices-aircraft, ships, automobiles, as well as civil and hydraulic engineering facilities-incorporate complex structural components, namely thin-walled curved pipes, which are widely used in engineering practice. During operation, such pipes typically interact with a liquid or gaseous medium and are subjected to various dynamic loads. Dynamic problems are particularly relevant in modern robotics for flexible jointless manipulators, which, due to their structural features, are highly sensitive to external loads [1–3]. Large-diameter thin-walled pipelines are essentially thin shells of either cylindrical or toroidal type. Therefore, when analyzing such pipelines, including their dynamic response, it is necessary to employ thin-shell theory and to explicitly account for the effect of internal pressure. For straight pipeline segments treated as cylindrical shells, these problems have been studied in sufficient detail in works [4,5,9] based on the theory of ideal-fluid flow; in those studies, the hydrodynamic pressure was determined in cylindrical coordinates as combinations of modified Bessel functions. At the same time, curved segments of large-diameter thin-walled pipelines-belonging to one of the most geometrically complex shell types, namely toroidal shells-are the most vulnerable components under operating conditions. However, the natural vibrations of such segments with the fluid flow taken into account have not been investigated adequately, largely due to difficulties in evaluating the magnitude of the fluid's hydrodynamic pressure.

Materials and Methods

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Problem statement. The considered section of the pipeline is represented as a section of toroidal shell with a radius R of the longitudinal axis passing through the centers of gravity of its cross-sections. The cross-sections are circular with the radius of the midline of the cross-section r, the thickness of the shell h, and the Kirchhoff-Love hypothesis is also valid. The end sections of the shell are assumed to be hinged. Inside the shell, a perfect incompressible fluid with density $p_0 = conts$ flows with velocity $U = const$. Toroidal shells with a middle surface are considered in toroidal curvilinear coordinates β, θ, where β is the central angle of the torus, and θ is the angle in the cross-section of the shell $(0 \quad \theta \quad 2\pi)$. The differential equation of motion of a curvilinear section of a pipeline with a stationary fluid flow, written in the displacements u, v, w, W_y, ϑ in toroidal coordinates β, θ, taking into account the components of the X_i^* inertia forces, takes the form:

$$\frac{r^2}{R^2}\frac{\partial^3 u}{\partial\beta\partial\theta^2}\cos\theta + \frac{r^3}{R^3}\frac{\partial^3 u}{\partial\beta^3} - \frac{r^2}{R^2}\frac{\partial}{\partial\theta}\quad\frac{\partial u}{\partial\beta}\sin\theta\quad + \frac{r^3}{R^3}\frac{\partial^2 W_y}{\partial\beta^2} + \frac{r^2}{R^2}$$

$$\frac{\partial}{\partial\theta}\quad\frac{\partial}{\partial\theta}\left(W_y\cos\theta\right) - W_y\sin\theta\quad + \frac{h^2}{r^2 12\left(1-\nu^2\right)}\frac{\partial^3}{\partial\theta^3}\quad\frac{\partial^2\vartheta}{\partial\theta^2} + \vartheta\quad = \frac{r^3}{ER}p\frac{\partial}{\partial\beta}\quad\frac{\partial^2 u}{\partial t^2}\quad -$$

$$-\frac{r^2}{E}p\frac{\partial}{\partial\theta}\quad\frac{\partial^2 v}{\partial t^2}\quad + \frac{r}{Eh}p_0\frac{\partial^3\vartheta}{\partial\theta^3} - \frac{r^2}{E}p\frac{\partial^4 w}{\partial\theta^2\partial t^2} + \frac{r}{Eh}\frac{\partial^2}{\partial\theta^2}\left(p_{ж}\right). \qquad (1)$$

Here, in the expression for the normal component of the X_3^* inertia forces, the internal pressure on the pipe wall according to [6] is represented as the sum $p = p_0 + p_{ж}$, $p_0 = const$ - constant hydrostatic pressure; $p_{ж}$ is the hydrodynamic pressure of the fluid flow in the curvilinear section of the pipeline, determined through the Legendre functions [6]. The last term of the right-hand side of equation (1) contains the derivative with respect to θ from the product $\frac{\partial^2\vartheta}{\partial\theta^2}X_3^*$.

$$\frac{r}{Eh}p_0\frac{\partial^3\vartheta}{\partial\theta^3} - \frac{r^2}{E}p\frac{\partial^4 w}{\partial\theta^2\partial t^2} + \frac{r}{Eh}\frac{\partial^2}{\partial\theta^2}\left(p_{ж}\right) \qquad (2)$$

By adding to the equation of motion (1) the relation of the semimomentless theory of shells [8] and using formula (2) for the hydrodynamic pressure $p_{ж}$, we obtain a complete system of equations of the set problem:

$$\frac{r^2}{R^2}\frac{\partial^3 u}{\partial\beta\partial\theta^2}\cos\theta + \frac{r^3}{R^3}\frac{\partial^3 u}{\partial\beta^3} - \frac{r^2}{R^2}\frac{\partial}{\partial\theta}\quad\frac{\partial u}{\partial\beta}\sin\theta\quad + \frac{r^3}{R^3}\frac{\partial^2 W_y}{\partial\beta^2} +$$

$$+\frac{r^2}{R^2}\frac{\partial}{\partial\theta}\quad\frac{\partial}{\partial\theta}\left(W_y\cos\theta\right) - W_y\sin\theta\quad + \frac{h^2}{r^2 12\left(1-\nu^2\right)}\frac{\partial^3}{\partial\theta^3}\quad\frac{\partial^2\vartheta}{\partial\theta^2} + \vartheta\quad =$$

$$= \frac{r^3}{ER}p\frac{\partial}{\partial\beta}\quad\frac{\partial^2 u}{\partial t^2}\quad - \frac{r^2}{E}p\frac{\partial}{\partial\theta}\quad\frac{\partial^2 v}{\partial t^2}\quad + \frac{r}{Eh}p_0\frac{\partial^3\vartheta}{\partial\theta^3} - \frac{r^2}{E}p\frac{\partial^4 w}{\partial\theta^2\partial t^2} -$$

$$-\frac{r^2}{Eh}p_0\Phi_n^*\quad\frac{\partial^4 w}{\partial\theta^2\partial t^2} + \frac{U^2}{Rr}\frac{\partial^4 w}{\partial\theta^2\partial\beta^2}\quad; \qquad (3)$$

-relations between displacements and deformations and the semimomentless theory of shells

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$$\frac{\partial v}{\partial \theta}+w=0,\quad \frac{r}{R}\frac{\partial v}{\partial \theta}+\frac{\partial u}{\partial \theta}=0,\quad \vartheta=\frac{\partial w}{\partial \theta}-v,\quad W_y=w\cos\theta-v\sin\theta \qquad (4)$$

It should be noted that the components of the displacements are dimensionless, therefore all terms of the system of equations (3), (4) are also dimensionless. To solve the system of equations (3), (4), we represent the normal component of the displacement $w(\beta,\theta,t)$ arising during the bending vibrations of the toroidal shell in the form that satisfies the boundary conditions at the edges of the shell:

$$w\Big|_{\substack{\beta=0\\ \beta=\pi}}=0,\qquad \frac{\partial^2 w}{\partial \beta^2}\Big|_{\substack{\beta=0\\ \beta=\pi}}=0 \qquad (5)$$

The displacements w satisfy the cyclicity condition along the circular coordinate θ:

$$w(\beta,\theta,t)=f(t)b_m\cos m\theta\sin n\beta \qquad (6)$$

where $f(t)$ is the time function $t, b_m=const$, m,n are the wave numbers that determine the forms of oscillations of the shell in the circular and longitudinal directions, respectively. From the relations (4) between the components of the displacement, at the value of w according to (6), we obtain expressions for the remaining components of the displacement and the angle of rotation:

$$u=-\frac{r}{R}\frac{n}{m^2}f(t)b_m\cos m\theta\cos n\beta,\ v=-\frac{1}{m}f(t)b_m\sin m\theta\sin n\beta, \qquad (7)$$

$$\vartheta=-\frac{m^2-1}{m}f(t)b_m\sin m\theta\sin n\beta,\ W_y=\frac{1}{2}\ b_{m+1}\frac{m+2}{m+1}+b_{m-1}\frac{m-2}{m-1}\ \cos m\theta\sin n\beta.$$

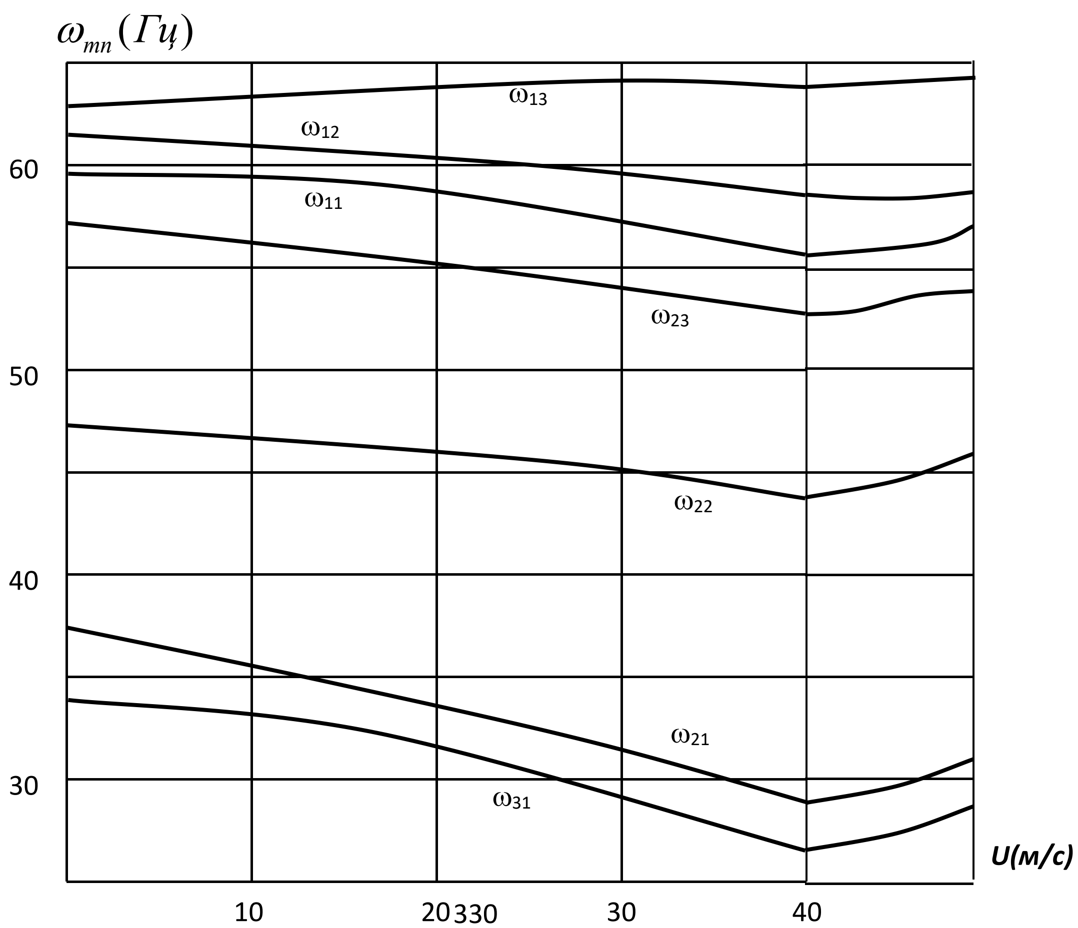

Fig.1.
Results
Fig.1. Frequency change own bending vibrations from the

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flowing liquid velocity (h=0.001).Substituting the expressions (6), (7) for the components of displacement and rotation angle into the shell motion equation (3) and calculating the partial derivatives for β and θ, we obtain the solving equation for unknown amplitude values of b_m and containing the time function $f(t)$and its second derivative for time $f\ (t)$. Since the solution of this homogeneous system of linear algebraic equations is different from zero, since the magnitudes of the amplitude values of the radial displacement of the middle surface of the shell $b_m \quad 0$, according to (6), the determinant of the coefficients of the homogeneous system must be equal to zero. Thus, the posed problem about the free oscillations of the curvilinear section of the pipeline with flowing liquid reduces to the problem with eigenvalues of the matrix A, where λ_i are the eigenvalues of the matrix, the role of which is played by the squares of the frequencies of natural oscillations ω_i^2. The frequency and forms of pipelines' natural vibrations, as well as the boundary conditions at the ends of a section of a curved pipe, are described taking into account their real fastenings. These conditions can be symmetrical and asymmetrical. For each type of fastening, its own beam functions are selected [6]. The flow velocity U, which varies within the range of real velocities flowing in the fluid pipelines (up to $25\frac{M}{c}$), has little effect on the natural vibration frequencies of the curvilinear sections of the steel pipeline for all the investigated shell-shaped vibrations ($m = 1,2,3$,4 at $n = 1,2,3$). The oscillation frequencies ω_{mn} decrease when the flow velocity increases from 0 to 25 $\frac{M}{c}$ by no more than 10%. For each of the considered sections of the pipeline, the highest natural vibration frequencies are for the first form ω_{1n} at $m = 1$.

Discussion

In the absence of deformation of the pipe's cross-sectional contour, the pipe oscillates like a tubular beam. For the dynamic calculation of the pipeline, the most important are the shell-shaped vibrations (at $m = 2$ and 3), corresponding to the deformed contour of the pipe's cross-section. With increasing curvature of the pipeline section, i.e., the $\frac{r}{R}$ ratio, at constant relative thickness ($\frac{h}{r} = const$), the frequency of natural bending vibrations ω_{mn} increases. Also, with increasing relative thickness ($\frac{h}{r}$, with constant pipe curvature), the natural frequency of bending vibrations increases. Thus, the greater the curvature of the pipe, the more rigid it becomes, and the thicker the pipe wall, the more rigid it becomes. This is also evident from the graphs in Figure 1, where the monotonically increasing dependence of the frequency ω_{21} on the shape of the oscillations is shown. At $m = 2$, the curvature parameter of the pipeline section μ and $\frac{h}{r}$ significantly affect the natural frequency of oscillations. The smaller the curvature of the pipe and the thinner its wall, the lower the frequency of its natural oscillations ω_{mn} for practically all forms. At a fluid velocity of 40 $\quad U \quad$ 50, the minimum frequency value ω_{mn} was determined. At these values of the curvilinear shell frequencies, the forces and deformations reach their maximum values.

Conclusion

1. The problem of determining the frequency and forms of natural bending vibrations in the curvature plane of curvilinear sections of thin-walled pipelines of large diameter (toroidal shells) with a stationary flow of ideal incompressible fluid has been solved.
2. Taking into account the internal working pressure, the pressure of the liquid on the wall is represented as the sum of the hydrostatic and hydrodynamic components; the hydrodynamic pressure for toroidal geometry is expressed through Lejandr functions, which allowed us to close the system of equations of motion.
3. Frequencies and forms of natural bending vibrations in the plane of curvature for curvilinear sections of thin-walled pipelines of large diameter, represented by toroidal shells, in the presence of flowing fluid, have been determined.
4. A practical methodology for using fundamental beam functions for calculating the frequency and shape of natural bending vibrations of curvilinear sections of pipelines with flowing liquid, taking into account internal working hydrostatic and hydrodynamic pressure, applicable to various boundary conditions and different values of the central angle α , has been proposed and substantiated.

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