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DERIVATION OF THE DISTANCE BETWEEN PARALLEL PLANES

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Abstract: This paper explores the method for determining the distance between two parallel planes. The formula for calculating the distance, its derivation, and geometric properties are discussed. The distance between two parallel planes is the shortest distance measured along a perpendicular line from one plane to the other. Through the formulas provided, examples are presented to explain how the distance between two parallel planes is calculated. This concept is widely applied in mathematics and fields such as physics, aiding in solving both theoretical and practical problems.

Key words: Two planes, parallel planes, distance, point**.**

Introduction

In geometry, the concept of distance between planes holds significant theoretical and practical value. In particular, calculating the distance between two parallel planes arises frequently in various scientific and engineering disciplines, including mathematics, physics, engineering design, and computer graphics. Parallel planes are defined as planes that do not intersect and share the same normal vector direction, making the shortest distance between them a constant. Although the formula for finding this distance appears straightforward, it incorporates essential geometric concepts such as plane equations, normal vectors, and the perpendicular distance from a point to a plane. In this article, we will explore the mathematical formulation for determining the distance between two parallel planes, provide a derivation of the formula, and discuss its practical applications through illustrative examples.[1-4].

Literature Review:

The problem of finding the distance between parallel planes is one of the fundamental topics in geometry. Numerous studies and scholarly works have mathematically analyzed this problem, proposing various formulas and approaches.

1. **Mirzoev, I. (2015).** Geometry: Theoretical Foundations and Practical Applications. Tashkent: Science and Technology.
This source provides detailed insights into planes and their position in space. Methods for calculating the distance between two parallel planes are presented, including theoretical approaches related to normal vectors and perpendicular distances.
2. **Sharifov, A., & Rakhimov, F. (2018).** Geometric Problems and Their Applications in Physics. Tashkent: National University of Uzbekistan.
In this study, Sharifov and Rakhimov discuss the calculation of distances between planes and their applications in physics, particularly in electrostatics. The role of the distance between parallel planes in electrostatic fields and its calculation methods are explained.
3. **G‘ulomov, B. (2020).** Mathematical Models and Geometry. Tashkent: O‘qituvchi.
This book mathematically proves the formula for calculating the distance between parallel planes and provides examples of its application. The book includes various methodologies to help understand the formula and its proof.

The literature review indicates that the problem of finding the distance between parallel planes is widely used across different fields of mathematics, and it holds practical significance in various sciences, including physics and engineering. [4-6].

Results and Discussion

Let's say that we take mutually parallel planes α_1 and α_2 in space.

$$\alpha_1\colon\ A_1x + B_1y + C_1z + D_1 = 0$$

$$\alpha_2\colon\ A_2x + B_2y + C_2z + D_2 = 0$$

If the point $M(x_1, y_1, z_1)$ is determined from the plane α_1, the perpendicular α_2 plane normal vector drawn from this point is parallel to $\vec{N} = (A_2, B_2, C_2)$. We draw a straight line parallel to the vector $\vec{N}$ from M. Its canonical and parametric equations are as follows:

$$\frac{(x - x_1)}{A_2} = \frac{y - y_1}{B_2} = \frac{z - z_1}{C_2} = t$$

$$\begin{cases} x = A_2t + x_1 \\ y = B_2t + y_1 \\ z = C_2t + z_1 \end{cases}$$

A straight line perpendicular to the α_2 plane intersects the plane at the point $K(x, y, z)$. [7-8].

We put the expression of x, y and z in the parametric equation through the parameter t into the plane equation α_2:

$$A_2(A_2t + x_1) + B_2(B_2t + y_1) + C_2(C_2t + z_1) + D_2 = 0$$

$$A_2^2t + A_2x_1 + B_2^2t + B_2y_1 + C_2^2t + C_2z_1 + D_2 = 0$$

$$t\left(A_2^2 + B_2^2 + C_2^2\right) + A_2x_1 + B_2y_1 + C_2z_1 + D_2 = 0$$

$$t = -\frac{A_2x_1 + B_2y_1 + C_2z_1 + D_2}{A_2^2 + B_2^2 + C_2^2}$$

$$d = |MK| = \sqrt{((x - x_1)^2 + (B_2 t)^2 + (z - z_1)^2)}$$

$$\begin{cases} x - x_1 = A_2 t \\ y - y_1 = B_2 t \\ z - z_1 = C_2 t \end{cases}$$

$$d = |MK| = \sqrt{(A_2 t)^2 + (B_2 t)^2 + (C_2 t)^2)}$$

$$d = |MK| = |t|\sqrt{(A_2^2 + B_2^2 + C_2^2)} = \left| \frac{A_2 x_1 + B_2 y_1 + C_2 z_1 + D_2}{\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

From this came the distance between the point and the plane.[8-12].

Now we make the formula for the distance from this point to the plane and the distance from the plane to the plane. The coordinates of the normal vector in the equation $A_1 x_1 + B_1 y_1 + C_1 z_1 + D_1 = 0$ in space cannot be equal to 0. [12-15].

From this

1) $A_1 \neq 0 \ \ x_1 = -\frac{(D_1 + C_1 + B_1)}{A_1}, y_1 = 1, z_1 = 1$,

2) $A_1 = 0 \ B_1 \neq 0 x_1 = 1, y_1 = -\frac{(D_1 + C_1 + A_1)}{B_1}$,

3) $z_1 = 1, A_1 = 0, B_1 = 0, C_1 \neq 0, x_1 = 1, y_1 = 1, z_1 = -\frac{D_1 + B_1 + A_1}{C_1}$.

If we put in the formula of the distance from the above point to the plane,

1) $x_1 = -\frac{(D_1 + C_1 + B_1)}{A_1}, y_1 = 1, z_1 = 1 \ \ A_1 \neq 0$;

$$d = \left| \frac{\begin{vmatrix} D_2 & D_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} B_2 & B_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} C_2 & C_1 \\ A_2 & A_1 \end{vmatrix}}{A_1 \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

2) $x_1 = 1, y_1 = -\frac{A_1 + C_1 + D_1}{B_1}, z_1 = 1, B_1 \neq 0$;

$$d = \left| \frac{\begin{vmatrix} A_2 & A_1 \\ B_2 & B_1 \end{vmatrix} + \begin{vmatrix} C_2 & C_1 \\ B_2 & B_1 \end{vmatrix} + \begin{vmatrix} D_2 & D_1 \\ B_2 & B_1 \end{vmatrix}}{B_1 \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

3) $x_1 = 1, y_1 = 1, \ z_1 = -\frac{D_1 + B_1 + A_1}{C_1}, C_1 \neq 0$;

$$d = \left| \frac{\begin{vmatrix} A_2 & A_1 \\ C_2 & C_1 \end{vmatrix} + \begin{vmatrix} B_2 & B_1 \\ C_2 & C_1 \end{vmatrix} + \begin{vmatrix} D_2 & D_1 \\ C_2 & C_2 \end{vmatrix}}{C_1 \sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

Example: Find the distance between two parallel planes

$x + 2y - 2z + 2 = 0, \quad 3x + 6y - 6z - 4 = 0.$

If we find the distance between two planes in space using coefficients:

$$d = \left| \frac{\begin{vmatrix} D_2 & D_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} B_2 & B_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} C_2 & C_1 \\ A_2 & A_1 \end{vmatrix}}{A_1\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

$$d = \left| \frac{\begin{vmatrix} -4 & 2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} 6 & 2 \\ 3 & 1 \end{vmatrix} + \begin{vmatrix} -6 & -2 \\ 3 & 1 \end{vmatrix}}{1\sqrt{(3)^2+(6)^2+(6)^2}} \right| = \left| \frac{(-4-6)+(6-6)+(-6+6)}{9} \right| = \frac{10}{9}.$$. [15-17].

Conclusions

General Outcome

Beyond the classical formula

$$d = \left| \frac{\begin{vmatrix} D_2 & D_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} B_2 & B_1 \\ A_2 & A_1 \end{vmatrix} + \begin{vmatrix} C_2 & C_1 \\ A_2 & A_1 \end{vmatrix}}{A_1\sqrt{A_2^2 + B_2^2 + C_2^2}} \right|$$

this work presented a parametric derivation based on constructing the perpendicular line from a point on one plane to the other.

Special Cases

When certain coefficients of the first plane vanish (A_1=0, B_1=0, or C_1=0), three distinct point-selection strategies were developed. These ensure the method applies to any pair of parallel planes.

Practical and Theoretical Significance

- **Practical**: The approach is well-suited for accurately computing distances in electrostatics, engineering structures, and computer graphics where complex parallel plane configurations arise.
- **Theoretical**: The parametric proof reinforces fundamental geometric concepts—normal vectors, perpendicular projections, and parametric line equations.

Future Work

- Extending to distances between hyperplanes in higher-dimensional spaces.
- Enhancing computational accuracy and efficiency of the parametric method using numerical techniques.

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