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ORDINARY DIFFERENTIAL EQUATIONS WITH DELAYED ARGUMENTS

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Abstract: This article presents proposals and recommendations regarding ordinary differential equations with delayed arguments and methods for their solution.

Keywords: delayed argument, ordinary differential equation, function

In recent years, interest in differential equations involving unknown functions with piecewise continuous and delayed arguments has been increasing. This is due to the fact that the mathematical modeling of many heat conduction and diffusion processes leads to problems for differential equations in which the function appears at certain values. Typically, such equations are referred to as differential equations with delayed arguments. Therefore, in this study, we examine several problems for first-order differential equations with delayed arguments.

Definition. The differential equation

$$y'(x)=f(x,y(x-\alpha),\, y(\beta x),\, \lambda) \qquad (1)$$

is called a firs-order differential equation with delayed arguments. Here, $\lambda-$ is a parameter, α, β are constants, $x-$is the ordinary argument,
$(x-\alpha)$, and $\beta x-$ are the delayed arguments, and $y(x)-$is the unknown function.

We seek the solution of the given equation (1) in the form of the following functional series:

$$y(x)=y_0(x)+\lambda y_1(x)+\lambda^2 y_2(x)+\lambda^3 y_3(x)+\ldots+\lambda^n y_n(x)+\ldots \qquad (2)$$

Here, $y_0, y_1, y_2\ldots$ are unknown functions, which should be chosen in such a way that the series (2) becomes the solution of equation (1).

For this purpose, we consider (2) as the solution of the equation and substitute it into equation (1). The unknown functions $y_0, y_1, y_2\ldots$ are determined by equating the coefficients of like powers of λ\lambdaλ on both sides of the resulting identity.

Here, we assume that the functional series (2) is uniformly convergent on the interval under consideration; therefore, it can be differentiated and integrated term by term.

Example 1.$2xy'(x)=2x+y(x)-y\left(\frac{x}{5}\right)$ Find the general solution of the equation:

Solution: Multiply both sides of the given differential equation by $2x$ and rewrite it in the following form: $y'(x)=1+\frac{1}{2x}\left[y(x)-y\left(\frac{x}{5}\right)\right]$, $\lambda=\frac{1}{2}, \beta=\frac{1}{5}$

This equation is a differential equation with delayed arguments, and we seek its solution in the form of the functional series (2):

$$y_0{}'(x)+\lambda y_1{}'(x)+\lambda^2 y_2{}'(x)+\lambda^3 y_3{}'(x)+\ldots+\lambda^n y_n{}'(x)+\ldots=1+$$

$$+\frac{\lambda}{x}[y_0(x)+\lambda y_1(x)+\lambda^2 y_2(x)+\ldots]-\frac{\lambda}{x}[y_0(\beta x)+\lambda y_1(\beta x)+\lambda^2 y_2(\beta x)+\ldots]$$

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$\lambda^n (n=0,1,2,...)$ By equating the corresponding coefficients, we determine the functions $y_n(x)$.

$$y_0'(x)=1, \; y_0(x)=x+C_0$$

$$y'(x)=ay(x)+by(x-\omega), x\geq 0, >0, y(x)\equiv\varphi(x), -\omega\leq x\leq 0, \qquad (3)$$

Here, $\varphi(x)$ is a continuous and differentiable function.

Theorem. The function $y(x)$ satisfying equation (2) and the condition $y(x)\equiv\varphi(x), -\omega\leq x\leq 0$is determined as follows:

$$y(x)=e^{a(x+\omega)}e_{\omega}^{b_1 x}\varphi(-\omega)+\int_{-\omega}^{0} e^{a(x-\omega)}e_{-\omega}^{b_1(x-\omega-s)}[\varphi'(s)-a\varphi(s)]ds \qquad (4)$$

Definition. The function e_{ω}^{bx} is called a function with a delayed argument.

$$e_{\omega}^{bx}=\begin{cases} 0, & -\infty<x<-\omega \\ 1, & -\omega\leq x<0 \\ 1+b\frac{x}{1!}, & 0\leq x<\omega \\ 1+b\frac{x}{1!}+b^2\frac{(x-\omega)^2}{2!}, & \omega\leq x<2\omega \\ \dots\dots\dots & \\ 1+b\frac{x}{1!}+\ldots+b^k\frac{[x-(k-1)\omega]^k}{k!}, & (k-1)\omega\leq x<kx, \\ \dots\dots\dots & \end{cases} \qquad (5)$$

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