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INTEGRAL EQUATIONS

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Abstract: This article presents the fundamental concepts of the theory of integral equations, the classification of integral equations, and the main analytical and numerical methods for solving them. Methods for finding solutions to Fredholm- and Volterra-type integral equations are discussed, in particular, the successive approximation method, the resolvent kernel, and the degenerate kernel method. The presented results serve to highlight the practical applications of the theory of integral equations..

Keywords: integral equation, Fredholm equation, Volterra equation, successive approximation method, resolvent, numerical methods.

Introduction. Integral equations are an important part of mathematical analysis and applied mathematics, appearing in numerous fields such as physics, mechanics, electrical engineering, economics, and biology. Many boundary value and initial value problems are reduced to integral equations or are equivalent to them. Therefore, the study of integral equations and the development of effective methods for their solution remain pressing scientific tasks.

The theory of integral equations developed actively at the end of the 19th and the beginning of the 20th centuries. In particular, the classes of integral equations proposed by V. Volterra and I. Fredholm played a significant role in the advancement of this field. Today, integral equations are widely used not only in pure mathematics but also in modeling numerous practical problems.

Types of Integral Equations: In general, an integral equation can be written in the following form:

$$\varphi(x)=f(x)+\lambda\int_a^b K(x,t)\varphi(t)dt \qquad (1)$$

Here, $\varphi(x)-$ is the unknown, $f(x)-$ is a given function, $K(x,t)-$ is the kernel of the integral, and, $\lambda-$ a parameter.

Integral equations are classified according to several criteria as follows:

- **Fredholm integral equations**, if the limits of integration are fixed;
- **Volterra integral equations**, if one of the limits of integration is variable;
- **Linear and nonlinear integral equations**;
- **Homogeneous and nonhomogeneous integral equations**.

The following methods are primarily used for solving integral equations:

a) Successive approximation method;
b) Quadrature formulas method;
c) Galerkin method.

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Some Applications of Integral Equations: The theory of integral equations is widely used in various fields of science and engineering and serves as an important tool for the mathematical modeling of complex processes. Below are the main application areas of integral equations**.**

Applications in Physics and Mechanics: In physics, integral equations arise in potential theory, electrostatics, quantum mechanics, and heat conduction problems. For example, boundary value problems formulated for Laplace and Poisson equations are often reduced to Fredholm- or Volterra-type integral equations. In mechanics, integral equations are used to solve problems in elasticity theory, vibration processes, and aerodynamics.

Applications in Technology and Engineering: Integral equations play an important role in electrical and radio engineering for the analysis of signals and systems. In particular, the relationships between currents and voltages in electrical circuits are expressed through integral equations. In automatic control systems, integral equations are also used to determine the system's state and to analyze its stability.

Applications in Economics and Financial Modeling: In modeling economic processes, integral equations appear in problems related to capital accumulation, investment flows, and resource allocation. Some macroeconomic models are expressed using Volterra-type integral equations, which allow for the consideration of time delays in the system.

Applications in Biology and Medicine: In biology, integral equations are used to study population dynamics, the spread of epidemics, and cell growth. Models with age- or time-dependent delays are particularly conveniently expressed using integral equations. In medicine, integral equations play an important role in pharmacokinetics and the processing of biological signals**.**

Applications in Mathematical Physics and Other Fields: Many problems in mathematical physics, including boundary value problems and the theory of integral operators, are closely related to integral equations. Additionally, integral equations are effectively applied in probability theory, statistical physics, and optimization problems.

Conclusion: This article provides a detailed overview of the fundamental concepts, types, and analytical and numerical solution methods for Fredholm- and Volterra-type integral equations. Information on the successive approximation method and resolvent-based numerical solution methods is also presented.

Furthermore, the applications of integral equations in various fields were analyzed. It was shown that in practical areas such as physics, mechanics, electrical engineering, economics, biology, and medicine, integral equations serve as an important tool for modeling, forecasting, and analyzing processes. This demonstrates the strong connection between the theory of integral equations and applied mathematical modeling.

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