JOURNAL OF MULTIDISCIPLINARY SCIENCES AND INNOVATIONS
VOLUME 04, ISSUE 11
MOPNTHLY JOURNALS
ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08

CAUCHY PROBLEM FOR THE RICCATI EQUATION WITH PIECEWISE CONTINUOUS ARGUMENT

Department of Mathematics, Navoiy State University
Associate Professor **S.Kh. Kholikov**
Master's Student, Asia International University, **M.N. Keldiyorova**

Abstract: This article addresses the Cauchy problem for the Riccati equation, one of the important classes of first-order nonlinear ordinary differential equations. The Cauchy problem is formulated for a Riccati differential equation with a piecewise continuous argument, and the existence of its solution is analyzed. The obtained results contribute to the development of the theory of differential equations with piecewise continuous arguments.

Keywords: Riccati equation, Cauchy problem, first-order nonlinear ordinary differential equations, piecewise continuous argument

Introduction. First-order nonlinear ordinary differential equations constitute an important part of mathematical analysis and the theory of differential equations. Such equations frequently arise in the mathematical modeling of various natural and social processes, and their solutions play a crucial role in determining the laws governing the development of these processes. Due to their nonlinearity, these equations are significantly more complex than linear differential equations, requiring specialized theoretical and practical approaches for their study.

Among first-order nonlinear differential equations, the Riccati equation holds a special place. This equation is related to many other types of equations and finds wide applications in second-order linear differential equations, optimal control problems, stability theory, and variational calculus. While the classical form of the Riccati equation has been studied extensively, its generalized forms remain relevant today.

In recent years, interest in differential equations with non-continuous arguments—particularly those with piecewise continuous or discrete characteristics—has increased significantly. Such equations model real processes that involve jumps, delays, or anticipatory effects over time. Examples include impulsive effects in technical systems, periodic decision-making in economic models, and stepwise population development in biological systems. Riccati equations with piecewise continuous arguments serve as convenient mathematical models for describing these processes.

In these equations, the behavior of the solution depends not only on the function itself but also strongly on the continuity properties of the argument function. Therefore, for such equations, the proper formulation of the Cauchy problem, the existence and uniqueness of solutions, and their dependence on initial conditions constitute important scientific issues.

In this study, we examine the Cauchy problem for the Riccati equation with a piecewise continuous argument, which is one of the significant classes of first-order nonlinear ordinary differential equations. The main goal of this research is to formulate this problem correctly and analyze the existence of its solution.

Problem Statement:

$$y'(x)+ay^2([x])+by([x])+c=0 \quad (1)$$

$$y(0)=y_0 \quad (2)$$

In the problem (1)-(2) $a,b,c\in R$, and $y(x)-$ is a continuous function..

$$n\le x<n+1,\ \ y'(x)=-ay^2(n)-by(n)-c \quad (3)$$

Equation (3) is a first-order ordinary differential equation with separable variables.

$$y_n=y(n),\ y_{n+1}=y_n-(ay_n^2+by_n+c) \quad (4)$$

$$C_n=-(ay_n^2+by_n+c),\ \ y=C_n(x-n)+y_n \quad (5)$$

Properties of the Solution:

- The solution is continuous on the entire interval $[0,+\infty)$;
- On each subinterval $[n,n+1)$, it is continuous and differentiable;
- The solution depends on the initial value y_0 and the coefficients a,b,c;
- The solution of the problem is connected to a discrete recurrent system.

Conclusion: In this article, the Cauchy problem for the Riccati equation, one of the important classes of first-order nonlinear ordinary differential equations, was investigated in the case of a piecewise continuous argument. The involvement of the floor function as the argument allowed the equation to be considered as a generalized form of the classical Riccati equation.

References

1. J. K. Hale, Theory of Functional Differential Equations (Springer-Verlag, New York-Heidelberg-Berlin, 1977).
2. O. Arino and I. Ciyöri, Necessary and sufficient condition for oscillation of a neutral differential system with several delays, J. Differential Equations 81 (1989), 98–105.
3. A. R. Aftabizadeh, J. Wiener, and J.-M. Xu, Oscillatory and periodic solutions of delay differential equations with piecewise constant argument, Proc. Amer. Math. Soc. 99 (1987), 673–679.
4. J. Wiener and K. L. Cooke, Oscillations in systems of differential equations with piecewise constant argument, J. Math. Anal. Appl. 137 (1989), 221–239.
5. J. Wiener and L. Debnath, Partial differential equations with piecewise constant delay, Int. J. Math. and Math. Sci. 14 (1991), 485–496.
6. Y. Kitamura and T. Kusano, Oscillation of first-order nonlinear differential equations with deviating argument, Proc. Amer. Math. Soc. 78 (1980), 64–68.
7. Huseyin Bereketoglua and Mehtap Lafcia, Behavior of the solutions of a partial differential equation with a piecewise constant argument, Filomat 31:19 (2017), 5931–5943. https://doi.org/10.2298/FIL1719931B
8. Kh. Kh. Turdiev, M. O. Rajabova, S. X. Xolikov, B. T. Karamatov, Direct and inverse coefficient problems for a fractional diffusion-wave equation with Riemann–Liouville derivative in time, Izvestiya Zavedeniy. Matematika, 2025, No. 11, https://doi.org/10.26907/0021-3446-2025-11-70-82

JOURNAL OF
MULTIDISCIPLINARY
SCIENCES AND INNOVATIONS

VOLUME 04, ISSUE 11
MOPNTHLY JOURNALS

ISSN NUMBER: 2751-4390
IMPACT FACTOR: 9,08