

**CENTRAL TENSION AND COMPRESSION OF RODS**

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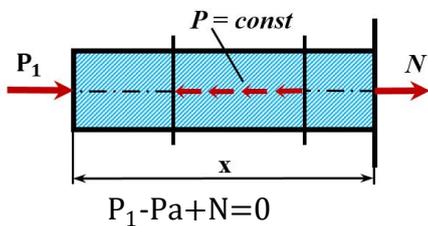
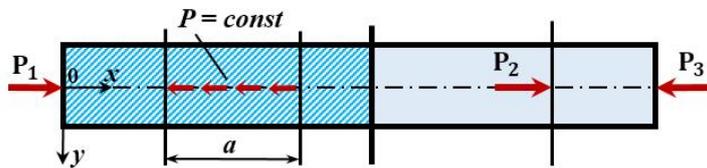
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Central tension or compression of a beam occurs when all the loads applied to the beam, or their resultant, are directed along its axis (axial loads). In this case, only normal stresses act in the cross-sections of the beam, which can be reduced to a single internal force — the axial force  $N$ . Given the known loads and support reactions, the axial force in the cross-sections of the beam can be determined statically using the method of sections (Fig. 1.1).



Equilibrium of the left part:

$$\sum x = 0$$

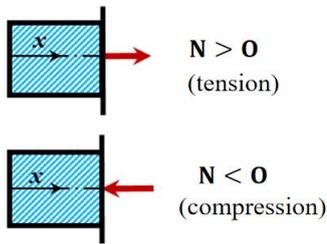
$$P_1 - Pa + N = 0$$

$$N = Pa - P_1$$

(Fig. 1.1). Determination of the axial force

Thus, the axial force in any cross-section of the beam is determined as the sum of the projections of all loads applied to one of the beam's segments onto its axis. Tensile axial forces are considered positive, while compressive forces are considered negative (Fig. 1.2).

In general, the axial force varies along the length of the beam. The following differential relationship exists between the axial force and the distributed axial load:



$$\frac{dN}{dx} = -P(x)$$

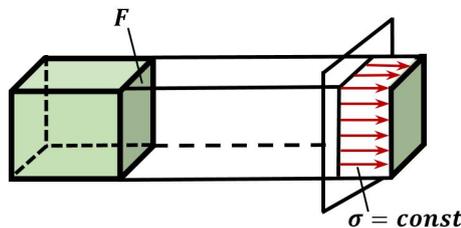
(Fig. 1.2). Sign of the axial force

This relationship allows determining the nature of the axial force variation depending on the type of distributed axial load.

Normal stresses in a beam under central tension and compression are assumed to be uniform across the cross-section. They are determined by the following formula:

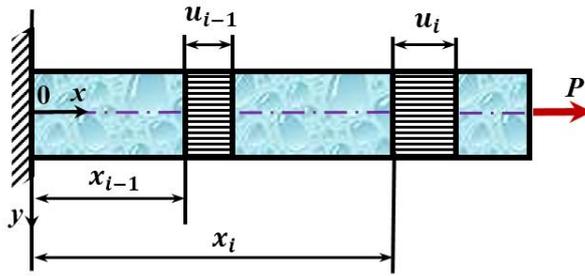
$$\sigma = \frac{N}{F}$$

where  $F$  is the cross-sectional area of the beam (Fig. 1.3).



(Fig. 1.3). Normal stresses

The deformation of a beam under central tension and compression is characterized by the axial displacements of its cross-sections (Fig. 1.4), which are related to each other by the following formula:



$$u_i = u_{i-1} + \Delta l_i \quad (1.3)$$

where  $\Delta l_i$  – is the absolute elongation or shortening of the beam segment between cross-sections  $x_i$  and  $x_{i-1}$ .

(Fig. 1.4). Axial displacements

The relative linear strains of the longitudinal fibers of a beam are related to the axial displacements by Cauchy's formula:

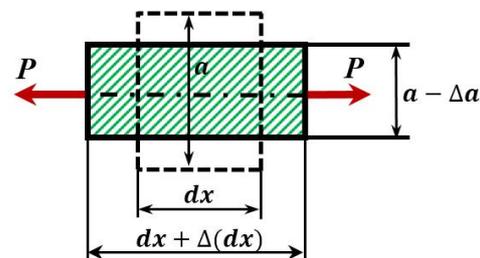
$$\epsilon_x = \epsilon = \frac{du}{dx} \quad (1.4)$$

The dimensions of the beam's cross-sections decrease under tension and increase under compression (Fig. 1.5). This phenomenon is referred to as lateral deformation and is characterized by Poisson's ratio:

$$\mu = \left| \frac{\epsilon'}{\epsilon} \right| \quad (1.5)$$

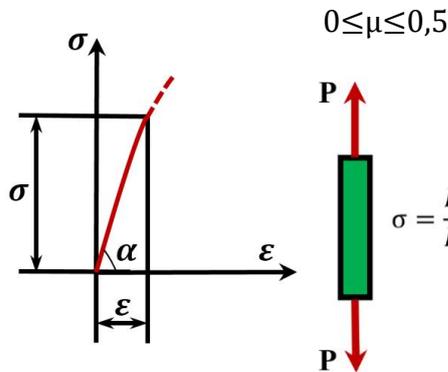
where  $\epsilon'$  and  $\epsilon$  - are the relative linear transverse and longitudinal strains;

$$\epsilon' = \frac{\Delta a}{a}, \quad \epsilon = \frac{\Delta(dx)}{dx} \quad (1.6)$$



(Fig. 1.5). Lateral deformation

The strains  $\varepsilon'$  and  $\varepsilon$  always have opposite signs. Poisson's ratio is determined experimentally and, for various materials, ranges within



For linearly elastic materials, the relationship between  $\sigma$  and  $\varepsilon$  is described by Hooke's law under tension and compression (Fig. 1.6):

$$\sigma = E \cdot \varepsilon \quad (1.7)$$

where  $E = \tan \alpha$  - the exact value depends on the grade of steel and testing conditions, but for engineering calculations it is usually taken as  $E = (2 \div 2,1) \cdot 10^5$  MPa.

(Fig. 1.6). Tension-compression diagram

The absolute elongation or shortening of a bar segment of length  $l$  is, in the general case, determined by the formula:

$$\Delta l = \int_l \frac{N}{EF} dx \quad (1.8)$$

where  $EF$  is the axial stiffness of the bar in tension or compression.

If the stiffness and the axial force are constant along the length of the bar, the absolute elongation or shortening is determined by a simpler formula:

when  $EF = \text{const}$ ,  $N = \text{const}$ , 
$$\Delta l = \frac{Nl}{EF} \quad (1.9)$$

If the axial force or the stiffness varies along the length of the bar, it is convenient to use the geometric meaning of the definite integral (1.8) for calculating  $\Delta l$ . For example, if  $E = \text{const}$ , formula (1.8) can be written in the following form:

$$\Delta l = \frac{1}{E} \int_l \frac{N}{F} dx = \frac{1}{E} \int_l \sigma dx = \frac{1}{E} \Omega_\sigma, \quad (1.10)$$

where  $\Omega_\sigma$  - the area of the  $\sigma$  diagram (epure) over a given segment, taken with the sign of the stresses into account.

For a bar with constant axial stiffness  $EF$  along its length, the axial tension and compression are described by the following differential equation:

$$EFu''(x) = -p(x) \quad (1.11)$$

The differential equation (1.11) together with relation (1.1) allow us to determine the variation of the axial force and axial displacement along the length of the bar, depending on the type of axial distributed load  $\rho(x)$ . For example, for the two most common cases, we have:

1. On segments where  $\rho = 0$ ,  $N = \text{const}$ ,  $U(x)$  - linear law.
2. On segments where  $\rho = \text{const}$ ,  $N$  - linear law,  $U(x)$  - quadratic parabola

The strain (deformation) potential energy of a bar of length  $l$  under axial tension or compression in the general case is determined by the formula:

$$U = \int_1 \frac{N^2}{2EF} dx \quad (1.12)$$

A bar subjected to tension or compression is usually called a rod. The corresponding design formulas are as follows:

Condition of strength

$$\sigma = \frac{N_{\text{load}}}{F} \leq R \quad (1.13)$$

Formula for selecting the cross-sectional area

$$F \geq F_{\text{sec}} = \frac{N_{\text{load}}}{R} \quad (1.14)$$

Formula for determining the load-carrying capacity

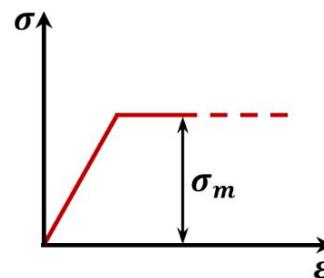
$$N_{\text{load}} \leq N_{\text{st}} = RF \quad (1.15)$$

Here  $N_{\text{load}}$  – axial force in the rod, calculated from the action of the design (calculated) loads, is denoted as:

$N_{\text{st}}$  – ultimate axial force according to the strength condition

$R$  – design (allowable) strength of the rod material in tension or compression

When determining the ultimate (failure) load for made of materials with plastic behavior, a simplified diagram (Fig. 1.7) is used. In this case, the failure axial rod under tension or compression is calculated by the



structures  
Prandtl  
force in a  
formula:

$$N_{\text{tn}} = \sigma_m \cdot F \quad (1.16)$$

Prandtl diagram

Where:  $\sigma_m$  - Yield strength of a material

(Fig. 1.7).

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