

SOLVING THE EQUATION OF VIBRATION OF A LATTICE USING THE METHOD OF SEPARATION OF VARIABLES (FOURIER METHOD)

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Abstract: This paper examines the nonlinear oscillations of a rigid body mounted on viscoelastic supports. The equations of motion of the system are derived from Lagrange's equations of the second kind for systems with a finite number of degrees of freedom. A solution method for the problem is developed, numerical results are obtained, and the influence of nonlinearity on displacement amplitudes is evaluated.

Keywords: nonlinear oscillations, rigid body, vibration, Lagrange equations, viscoelastic support, degree of freedom

1. Introduction

Issues related to dynamic vibration damping of mechanical systems are widely applied in machine dynamics. The use of passive vibration reduction methods is especially effective for objects exposed to steady periodic external forces. Developments are known that are associated with the creation of complex vibration reduction systems under multiple disturbances.

Theoretical approaches to solving vibration reduction problems based on structural methods of mathematical modeling are of certain interest. Within this framework, a mechanical oscillatory system is interpreted as a certain dynamic automatic control system. In this case, the structural diagram of the system acts as a structural analogue of the initial mathematical model, obtained in the form of a system of differential equations

2. Methods

The method of separation of variables (or Fourier method) that we will now consider serves as a model for solving many problems in mathematical physics. For example,

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} \quad (1)$$

of the equation

$$u(0, t) = 0, \quad (2)$$

$$u(l, t) = 0, \quad (3)$$

$$u(x, 0) = f(x), \quad (4)$$

$$\frac{\partial u}{\partial t} \Big|_{t=0} = \varphi(x) \quad (5)$$

We are required to find a solution satisfying the boundary conditions. We find the particular solution (not exactly equal to zero) of equation (1) satisfying the boundary conditions (2) and (3) in the form of a product of two functions $X(x)$ and $T(t)$, the first of which depends only on x , and the second only on t :

$$u(x, t) = X(x) T(t) \quad (6)$$

Putting these values into equation (1), we get:

$$X(x)T''(t) = a^2 X''(x)T(t) \quad (6)$$

and dividing the limits of this equality by $a^2 XT$,

$$\frac{T''}{a^2 T} = \frac{X''}{X} \quad (7)$$

we obtain the equality. The function on the left side of this equality does not depend on x , and the function on the right side does not depend on t . Equality (7) can be valid only if the left and right sides do not depend on either x or t , that is, if they are equal to a constant number. We denote it by $-\lambda$, where $\lambda > 0$ (later the case of $\lambda < 0$ will also be considered). Thus,

$$\frac{T''}{a^2 T} = \frac{X''}{X} = -\lambda$$

We create two equations from these equations:

$$X'' + \lambda X = 0, \quad (8)$$

$$T'' + a^2 \lambda T = 0 \quad (9)$$

The general solutions of these equations are as follows

$$X(x) = A \cos \sqrt{\lambda x} + B \sin \sqrt{\lambda x}, \quad (10)$$

$$T(t) = C \cos a\sqrt{\lambda t} + D \sin a\sqrt{\lambda t} \quad (11)$$

where A, B, C, D are arbitrary constants. Substituting the expressions $X(x)$ and $T(t)$ into equation (6), we obtain this equation:

$$u(x, t) = (A \cos \sqrt{\lambda x} + B \sin \sqrt{\lambda x})(C \cos a\sqrt{\lambda t} + D \sin a\sqrt{\lambda t})$$

Now we choose the constants A and B such that conditions (2) and (3) are satisfied. Since $T(t) \neq 0$ exists (otherwise $u(x, t) = 0$ exists, which contradicts the given condition), the function $X(x)$ must satisfy conditions (2) and (3), that is, it must be $X(x) = 0, X(l) = 0$. Substituting the values $x = 0$ and $x = l$ into equality (10), we obtain the following based on (2) and (3):

$$0 = A + B, \quad 0 = A \cos \sqrt{\lambda l} + B \sin \sqrt{\lambda l}.$$

From the first equation we find that $A = -B$. From the second equation

$$B \sin \sqrt{\lambda l} = 0$$

it turns out.

Note: If we had used the expression $-\lambda = k^2$ instead of $-\lambda$, equation (8) would have taken the form:

$$X'' - k^2 X = 0.$$

The general solution of this equation is as follows:

$$X = Ae^{kx} + Be^{-kx}.$$

A non-zero solution of such form (2) and (3) is marginal cannot satisfy the conditions.

$\sqrt{\lambda}$ knowing, using equality (11), such that we can write:

$$T(t) = C \cos \frac{an\pi}{l} t + D \sin \frac{an\pi}{l} t \quad (n = 1, 2, \dots) \quad (14)$$

For each value of n , and therefore for each λ , we substitute expressions (13) and (14) into equation (6) to obtain a solution of equation (1) satisfying boundary conditions (2) and (3). We denote this solution by $u_n(x, t)$:

$$u_n(x, t) = \sin \frac{n\pi}{l} x \left[C_n \cos \frac{an\pi}{l} t + D_n \sin \frac{an\pi}{l} t \right]. \quad (15)$$

3. Results and analysis

For each value of n , we can obtain our own constants C and D , and therefore we write them as C and D_n (the constant B is included in C and D_n). Since equation (1) is linear and homogeneous, the set of solutions is also a solution, and therefore

$$u(x, t) = \sum_{n=1}^{\infty} u_n(x, t)$$

or

$$u(x, t) = \sum_{n=1}^{\infty} \left[C_n \cos \frac{an\pi}{l} t + D_n \sin \frac{an\pi}{l} t \right] \sin \frac{n\pi}{l} x, \quad (16)$$

The function described by the series is also a solution of the differential equation (1) satisfying the boundary conditions (2) and (3). If the coefficients C_n and D_n are such that the series (16) and the series obtained by differentiating this series twice with respect to x and t are approximate, then it is clear that the series (16) is a solution of equation (1). The solution (16) must also satisfy the initial conditions (14) and (15). We achieve this by choosing the constants C_n and D_n . Putting $t = 0$ in equation (16), we obtain:

$$f(x) = \sum_{n=1}^{\infty} C_n \sin \frac{n\pi}{l} x. \quad (17)$$

If the function $f(x)$ can be expanded into a Fourier series on the interval $(0, l)$, then

$$C_n = \frac{2}{l} \int_0^l f(x) \sin \frac{n\pi}{l} x dx \quad (18)$$

If we assume that, condition (17) is satisfied. Now we differentiate the terms of equality (16) with respect to t and assume that $t=0$. From condition (15) we obtain this equality;

$$\varphi(x) = \sum_{n=1}^{\infty} D_n \frac{an\pi}{l} \sin \frac{n\pi}{l} x.$$

Let's determine the Fourier coefficients of this series:

$$D_n \frac{an\pi}{l} = \frac{2}{l} \int_0^l \varphi(x) \sin \frac{n\pi}{l} x dx$$

or

$$D_n = \frac{2}{an\pi} \int_0^l \varphi(x) \sin \frac{n\pi}{l} x dx \quad (19)$$

Thus, we have proved that the series (16), whose coefficients C_n and D_n are determined by formulas (18) and (19), if it can be differentiated twice, consists of the function $u(x, t)$ which is a solution of equation (I) satisfying the boundary and initial conditions (2)–(5). Note.

4. Conclusions

It is possible to solve the problem considered for the wave equation by another method and prove that the series (16) consists of a solution even if it is not possible to differentiate the series to the limit. Here, the function $f(x)$ must be a twice differentiable function, and $\varphi(x)$ must be a once differentiable function.

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