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DYNAMICS OF COMPOSITIONS OF LOTKA–VOLTERRA OPERATORS FOR SOME PARTIALLY ORIENTED GRAPHS IN A THREE-DIMENSIONAL SIMPLEX

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I. Introduction.

Let we have a simplex of three dimensions

$$S^3 = \quad x=(x_1,x_2,x_3,x_4): x_i \quad 0, \sum_{i=1}^{4} x_i = 1 \qquad R^4$$

and given quadratic operators of Lotka-Volterra type

$$V_1 : x_k = x_k \quad 1+\sum_{i=1}^{m} a_{ki}x_i \quad , and \; V_2 : x_k = x_k \quad 1+\sum_{i=1}^{m} b_{ki}x_i \quad ,k=\overline{1,m} \; (1)$$

on this simplex [1].

Definition 1 [1]. $(V_1 \circ V_2)(x) = V_1(V_2(x))$ or $(V_2 \circ V_1)(x) = V_2(V_1(x))$ a complex operator satisfying the equalities V_1 and V_2 is called a composition of operators.

Statement 1 [2]. According to (1), the composition of the operators V_1 and V_2 can be expressed in the form

$$W = V_1 \circ V_2 : x_k = x_k\left(1 + f_k\left(x_1,x_2,...,x_{k-1},x_{k+1},...,x_m\right)\right), k=\overline{1,m} \qquad (2)$$

Definition 2[1]. A point x satisfying the equality $W(x)=x$ is called a fixed point of W the operator and $Fix(W)=\{x \quad S^{m-1} : W(x)=x\}$ is defined as.

Definition 3[3]. A fixed point x is called an attractor if the trajectories of all sufficiently close points converge to this point. More precisely, if there is $\varepsilon>0$ a sufficiently small number of such $\varepsilon>0$ points, such that the equality $\lim_{k\to} W^k(y)=x$ holds for all y points in its vicinity ε , then this point is called an attractor.

Definition 4 [3]. . If the trajectories of all sufficiently close points diverge from this point, then x a fixed point is called a repeller. More precisely, if there is a sufficiently small number $\varepsilon > 0$ such that the relation $\lim_{k\to} W^k(y) \quad x$ holds for all y points around this ε , then this point is called a repeller.

Undirected, partially directed graphs and tournaments in S^3 are given in

Figure 1.

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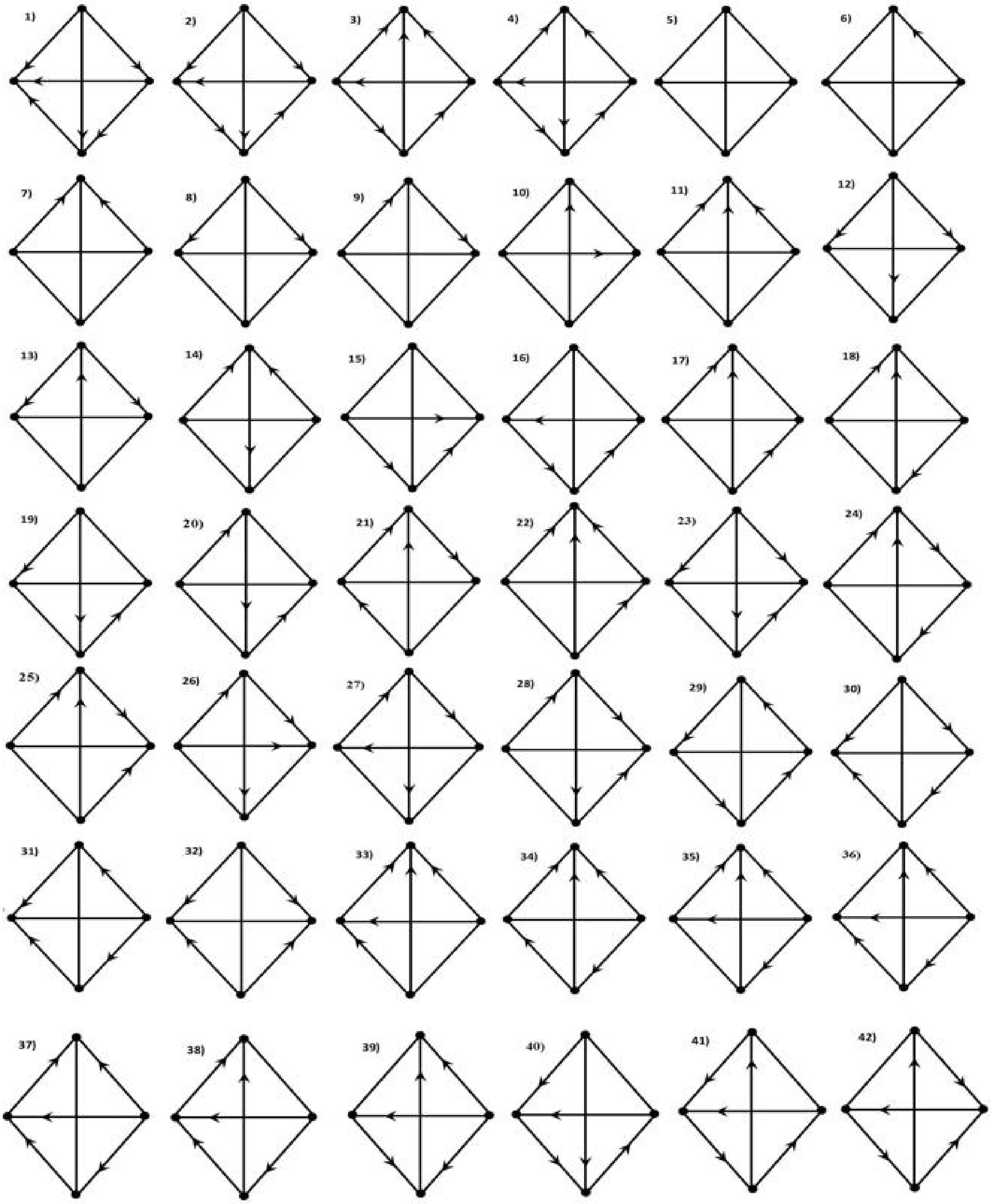

Figure 1. Partially directed graphs and tournaments in S^3.

Definition 5. The following matrix

$$J(W)=\begin{matrix} \frac{\partial x_1}{\partial x_1} & \frac{\partial x_1}{\partial x_2} & \cdots & \frac{\partial x_1}{\partial x_n} \\ \frac{\partial x_2}{\partial x_1} & \frac{\partial x_2}{\partial x_2} & \cdots & \frac{\partial x_2}{\partial x_n} \\ \cdots\cdots & \cdots\cdots & \cdots\cdots & \cdots\cdots \\ \frac{\partial x_n}{\partial x_1} & \frac{\partial x_n}{\partial x_n} & \cdots & \frac{\partial x_n}{\partial x_n} \end{matrix} \qquad (3)$$

consisting of the particular derivatives of a quadratic operator of Lotka–Volterra type is called the Jacobi matrix [4].

II. Main results.

Let that we are given the compositions of the Lotka–Volterra quadratic operators and their compositions corresponding to partially directed graphs (Figure 1) in S^3.

$$V_1: \begin{cases} x_1 = x_1(1+a_{12}x_2+a_{13}x_3+a_{14}x_4); \\ x_2 = x_2(1-a_{12}x_1); \\ x_3 = x_3(1-a_{13}x_1); \\ x_4 = x_4(1-a_{14}x_1); \end{cases} \qquad V_2: \begin{cases} x_1 = x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4); \\ x_2 = x_2(1+b_{12}x_1); \\ x_3 = x_3(1+b_{13}x_1); \\ x_4 = x_4(1+b_{14}x_1). \end{cases} \qquad (4)$$

Partially directed graphs corresponding to these operators are shown in Figure 2.

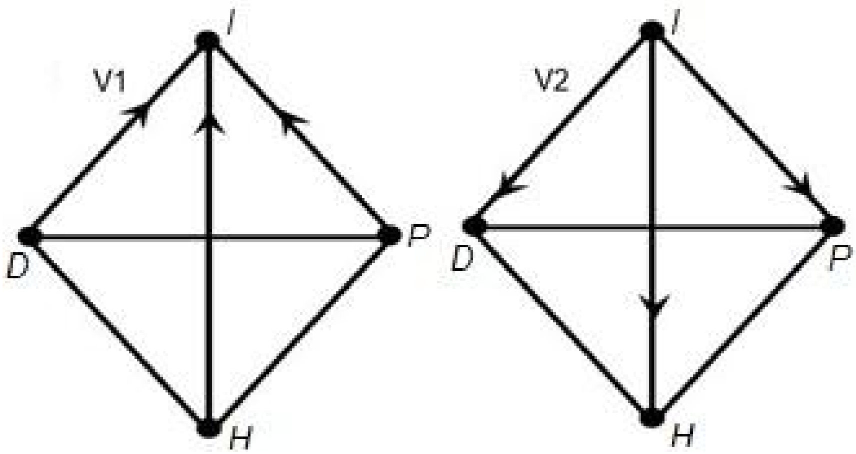

Figure 2. Partially directed graphs corresponding to operators V_1 *and* V_2

The composition of these operators looks like this:

$$V_1 \circ V_2: \begin{cases} x_1' = x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\left(1 + a_{12}x_2\left(1 + b_{12}x_1\right) + \right. \\ \left. + a_{13}x_3\left(1 + b_{13}x_1\right) + a_{14}x_4\left(1 + b_{14}x_1\right)\right); \\ x_2' = x_2\left(1 + b_{12}x_1\right)\left(1 - a_{12}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \\ x_3' = x_3\left(1 + b_{13}x_1\right)\left(1 - a_{13}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \\ x_4' = x_4\left(1 + b_{14}x_1\right)\left(1 - a_{14}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \end{cases} \tag{5}$$

Lemma 1. The following statements hold for the operator $V_1 \circ V_2$.

i) The vertices of the simplex I, P, H, and D, the points on the edges Γ_{IP}, Γ_{IH}, Γ_{ID}, and the fixed points of the composite operator Γ_{PHD} are all fixed points;

ii) All vertices of the operator $V_1 \circ V_2$ are attractor points;

iii) The fixed points on the edges of the operator $V_1 \circ V_2$ are saddle points.

Proof. By the definition of a fixed point, the solutions to equality $Vx = x$ represent the fixed points of the operator.

$$\begin{cases} x_1 = x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\left(1 + a_{12}x_2\left(1 + b_{12}x_1\right) + \right. \\ \left. + a_{13}x_3\left(1 + b_{13}x_1\right) + a_{14}x_4\left(1 + b_{14}x_1\right)\right); \\ x_2 = x_2\left(1 + b_{12}x_1\right)\left(1 - a_{12}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \\ x_3 = x_3\left(1 + b_{13}x_1\right)\left(1 - a_{13}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \\ x_4 = x_4\left(1 + b_{14}x_1\right)\left(1 - a_{14}x_1\left(1 - b_{12}x_2 - b_{13}x_3 - b_{14}x_4\right)\right); \\ x_1 + x_2 + x_3 + x_4 = 1. \end{cases} \tag{6}$$

The solution of the system of equations represents the fixed points of the given operator. The solution of equation (3.1.3) is equal to the vertices of the simplex $I(1;0;0;0)$, $P(0;1;0;0)$, $H(0;0;1;0)$, $D(0;0;0;1)$ and

$$O_1\left(\frac{(b_{12}-2)\sqrt{a_{12}} + \sqrt{a_{12}b_{12}^2 + 4b_{12}}}{2b_{12}\sqrt{a_{12}}}; \frac{(b_{12}+2)\sqrt{a_{12}} - \sqrt{a_{12}b_{12}^2 + 4b_{12}}}{2b_{12}\sqrt{a_{12}}}; 0; 0\right),$$

$$O_2\left(\frac{(b_{13}-2)\sqrt{a_{13}} + \sqrt{a_{13}b_{13}^2 + 4b_{13}}}{2b_{13}\sqrt{a_{13}}}; 0; \frac{(b_{13}+2)\sqrt{a_{13}} - \sqrt{a_{13}b_{13}^2 + 4b_{13}}}{2b_{13}\sqrt{a_{13}}}; 0\right),$$

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$$O_3\left(\frac{(b_{14}-2)\sqrt{a_{14}}+\sqrt{a_{14}b_{14}^2+4b_{14}}}{2b_{14}\sqrt{a_{14}}};0;0;\frac{(b_{14}+2)\sqrt{a_{14}}-\sqrt{a_{14}b_{14}^2+4b_{14}}}{2b_{14}\sqrt{a_{14}}}\right),$$

$$O_4(0;x_2;x_3;1-x_2-x_3).$$

points, representing the points corresponding to the Γ_{IP}, Γ_{IH}, Γ_{ID} edges and Γ_{PHD} sides of the simplex. The elements of the Jacobi matrix of this operator are:

$$\frac{\partial x_1}{\partial x_1}=(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big(1+a_{12}x_2(1+b_{12}x_1)+a_{13}x_3(1+b_{13}x_1)+a_{14}x_4(1+b_{14}x_1)\big)+$$
$$+x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)(a_{12}b_{12}x_2+a_{13}b_{13}x_3+a_{14}b_{14}x_4);$$

$$\frac{\partial x_1}{\partial x_2}=-\mathrm{b}_{12}x_1\big(1+a_{12}x_2(1+b_{12}x_1)+a_{13}x_3(1+b_{13}x_1)+a_{14}x_4(1+b_{14}x_1)\big)+$$
$$+a_{12}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)(1+b_{12}x_1);$$

$$\frac{\partial x_2}{\partial x_1}=b_{12}x_2\big(1-a_{12}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big)-a_{12}x_2(1+b_{12}x_1)(1-b_{12}x_2-b_{13}x_3-b_{14}x_4);$$

$$\frac{\partial x_2}{\partial x_2}=(1+b_{12}x_1)\big(1-a_{12}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big)+a_{12}b_{12}x_1x_2(1+b_{12}x_1);$$

$$\frac{\partial x_2}{\partial x_3}=a_{12}b_{13}x_1x_2(1+b_{12}x_1);$$

$$\frac{\partial x_2}{\partial x_4}=a_{12}b_{14}x_1x_2(1+b_{12}x_1);$$

$$\frac{\partial x_3}{\partial x_1}=b_{13}x_3\big(1-a_{13}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big)-a_{13}x_3(1+b_{13}x_1)(1-b_{12}x_2-b_{13}x_3-b_{14}x_4);$$

$$\frac{\partial x_3}{\partial x_2}=a_{13}b_{12}x_1x_3(1+b_{13}x_1);$$

$$\frac{\partial x_3}{\partial x_3}=(1+b_{13}x_1)\big(1-a_{13}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big)+a_{13}b_{13}x_1x_3(1+b_{13}x_1);$$

$$\frac{\partial x_3}{\partial x_4}=a_{13}b_{14}x_1x_3(1+b_{13}x_1);$$

$$\frac{\partial x_4}{\partial x_1}=b_{14}x_4\big(1-a_{14}x_1(1-b_{12}x_2-b_{13}x_3-b_{14}x_4)\big)-a_{14}x_4(1+b_{14}x_1)(1-b_{12}x_2-b_{13}x_3-b_{14}x_4);$$

$$\frac{\partial x_4}{\partial x_2}=a_{14}b_{12}x_1x_4(1+b_{14}x_1);$$

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$$\frac{\partial x_4}{\partial x_3} = a_{14} b_{13} x_1 x_4 \left(1 + b_{14} x_1\right);$$

$$\frac{\partial x_4}{\partial x_4} = \left(1 + b_{14} x_1\right)\left(1 - a_{14} x_1 \left(1 - b_{12} x_2 - b_{13} x_3 - b_{14} x_4\right)\right) + a_{14} b_{14} x_1 x_4 \left(1 + b_{14} x_1\right).$$

The elements of the Jacobian matrix for the simplex vertices are given in Table 1 .

Table 1 - *Elements of the Jacobian matrix for simplex vertices*

	I	***P***	***H***	***D***
$\frac{\partial x_1}{\partial x_1}$	1	$(1-b_{12})(1+a_{12})$	$(1-b_{13})(1+a_{13})$	$(1-b_{14})(1+a_{14})$
$\frac{\partial x_1}{\partial x_2}$	$-b_{12}+a_{12}(1+b_{12})$	0	0	0
$\frac{\partial x_1}{\partial x_3}$	$-b_{13}+a_{13}(1+b_{13})$	0	0	0
$\frac{\partial x_1}{\partial x_4}$	$-b_{14}+a_{14}(1+b_{14})$	0	0	0
$\frac{\partial x_2}{\partial x_1}$	0	$b_{12}-a_{12}(1-b_{12})$	0	0
$\frac{\partial x_2}{\partial x_2}$	$(1+b_{12})(1-a_{12})$	1	1	1
$\frac{\partial x_2}{\partial x_3}$	0	0	0	0
$\frac{\partial x_2}{\partial x_4}$	0	0	0	0
$\frac{\partial x_3}{\partial x_1}$	0	0	$b_{13}-a_{13}(1-b_{13})$	0
$\frac{\partial x_3}{\partial x_2}$	0	0	0	0
$\frac{\partial x_3}{\partial x_3}$	$(1+b_{13})(1-a_{13})$	1	1	1

$\frac{\partial x_3}{\partial x_4}$	0	0	0	0
$\frac{\partial x_4}{\partial x_1}$	0	0	0	$b_{14}-a_{14}(1-b_{14})$
$\frac{\partial x_4}{\partial x_2}$	0	0	0	0
$\frac{\partial x_4}{\partial x_3}$	0	0	0	0
$\frac{\partial x_4}{\partial x_4}$	$(1+b_{14})(1-a_{14})$	1	1	1

Table 2 below lists the eigenvalues of the fixed points I, P, H, and D of the composite operator $V_1 \circ V_2$.

Table 2. *Eigenvalues of fixed points I, P, H, and D of the composite operator* $V_1 \circ V_2$.

	I	***P***	***H***	***D***
λ_1	1	1	1	1
λ_2	$1-a_{12}b_{12}-a_{12}+b_{12}$	1	1	1
λ_3	$1-a_{13}b_{13}-a_{13}+b_{13}$	1	1	1
λ_4	$1-a_{14}b_{14}-a_{14}+b_{14}$	$1-a_{12}b_{12}+a_{12}-b_{12}$	$1-a_{13}b_{13}+a_{13}-b_{13}$	$1-a_{14}b_{14}+a_{14}-b_{14}$

According to Table 2, the absolute value of the eigenvalues of the fixed points I, P, H, and D of the composite operator $V_1 \circ V_2$ is $|\lambda_i| < 1, i = \overline{1,4}$. Therefore, the points I, P, H, and D are attractor points.

The eigenvalues of a point O_1 are as follows:

$$\lambda_1 = 1;$$

$$\lambda_2 = 1 - \frac{1}{4b_{13}\sqrt{a_{13}}}\left(b_{13}\sqrt{a_{13}} - \sqrt{b_{13}\left(a_{13}b_{13}+4\right)}\right)\left(-2b_{13}+a_{13}b_{13}+3\sqrt{a_{13}b_{13}\left(a_{13}b_{13}+4\right)}-2a_{13}\right)$$

$$\lambda_3 = 1 - \frac{1}{4b_{14}^2\sqrt{a_{14}^3}}\left(\left(b_{14}-2\right)\sqrt{a_{14}} + \sqrt{b_{14}\left(a_{14}b_{14}+4\right)}\right)\cdot$$

$$\bullet\left(\sqrt{b_{14}(a_{14}b_{14}+4)}\sqrt{a_{14}}\,(a_{14}b_{12}-a_{12}b_{14}+a_{12}b_{14})+a_{14}^2b_{12}b_{14}+a_{12}a_{14}b_{14}^2-a_{12}a_{14}b_{12}b_{14}-2a_{12}b_{12}b_{14}\right);$$

$$\lambda_4=2+\frac{1}{4b_{14}^2\sqrt{a_{14}^3}}\left((b_{14}-2)\sqrt{a_{14}}+\sqrt{b_{14}(a_{14}b_{14}+4)}\right)\bullet$$

The eigenvalues of a point O_2 are as follows:

$$\lambda_1=1;$$

$$\lambda_2=1-\frac{1}{4b_{13}\sqrt{a_{13}}}\left(b_{13}\sqrt{a_{13}}-\sqrt{b_{13}(a_{13}b_{13}+4)}\right)\left(-2b_{13}+a_{13}b_{13}+3\sqrt{a_{13}b_{13}(a_{13}b_{13}+4)}-2a_{13}\right);$$

$$\lambda_3=1-\frac{1}{4b_{13}^2\sqrt{a_{13}^3}}\left((b_{13}-2)\sqrt{a_{13}}+\sqrt{b_{13}(a_{13}b_{13}+4)}\right)\bullet$$

$$\bullet\left(\sqrt{b_{13}(a_{13}b_{13}+4)}\sqrt{a_{13}}\,(a_{13}b_{12}-a_{12}b_{13}+a_{12}b_{12})+a_{13}^2b_{12}b_{13}+a_{12}a_{13}b_{13}^2-a_{12}a_{13}b_{12}b_{13}-2a_{12}b_{12}b_{13}\right);$$

$$\lambda_4=2+\frac{\left(b_{13}\sqrt{a_{13}}-2\sqrt{a_{13}}+\sqrt{b_{13}(a_{13}b_{13}+4)}\right)\left(a_{13}b_{13}-2b_{13}+\sqrt{a_{13}b_{13}(a_{13}b_{13}+4)}\right)}{4\sqrt{a_{13}}b_{13}};$$

The eigenvalues of a point O_3 are as follows:

$$\lambda_1=1;$$

$$\lambda_2=1-\frac{1}{4b_{14}\sqrt{a_{14}}}\left(b_{14}\sqrt{a_{14}}-\sqrt{b_{14}(a_{14}b_{14}+4)}\right)\left(-2b_{14}+a_{14}b_{14}+3\sqrt{a_{14}b_{14}(a_{14}b_{14}+4)}-2a_{14}\right);$$

$$\lambda_3=1-\frac{1}{4b_{14}^2\sqrt{a_{14}^3}}\left((b_{14}-2)\sqrt{a_{14}}+\sqrt{b_{14}(a_{14}b_{14}+4)}\right)\bullet$$

$$\bullet\left(\sqrt{b_{14}(a_{14}b_{14}+4)}\sqrt{a_{14}}\,(a_{14}b_{12}-a_{12}b_{14}+a_{12}b_{14})+a_{14}^2b_{12}b_{14}+a_{12}a_{14}b_{14}^2-a_{12}a_{14}b_{12}b_{14}-2a_{12}b_{12}b_{14}\right);$$

$$\lambda_4=2+\frac{1}{4b_{14}^2\sqrt{a_{14}^3}}\left((b_{14}-2)\sqrt{a_{14}}+\sqrt{b_{14}(a_{14}b_{14}+4)}\right)\bullet$$

$$\bullet\left(\sqrt{b_{14}(a_{14}b_{14}+4)}\sqrt{a_{14}}\,(a_{14}b_{12}-a_{12}b_{14}+a_{12}b_{14})+a_{14}^2b_{12}b_{14}+a_{12}a_{14}b_{14}^2-a_{12}a_{14}b_{12}b_{14}-2a_{12}b_{12}b_{14}\right);$$

the lemma has been proven.

Since the proof of the results resulting from this lemma can be applied to the dynamics of compositions of quadratic operators of the Lotka–Volterra type corresponding to all S^3 partially oriented graphs and fully oriented tournaments given in Figure 1, we present the remaining lemmas without proof. Let us first consider the composition of quadratic operators of the Lotka–Volterra type corresponding to fully oriented tournaments and unoriented graphs. Suppose that the following operators are given:

I)

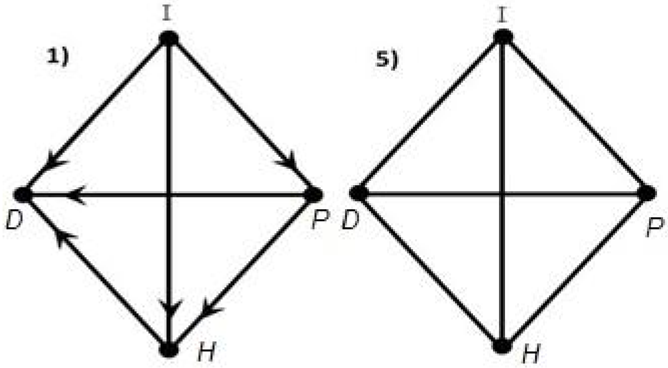

Figure 3. *Fully directed tournament and undirected graph*

$$1): \begin{array}{l} x_1 = x_1(1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4); \\ x_2 = x_2(1 + a_{12}x_1 - a_{23}x_3 - a_{24}x_4); \\ x_3 = x_3(1 + a_{13}x_1 + a_{23}x_2 - a_{34}x_4); \\ x_4 = x_4(1 + a_{14}x_1 + a_{24}x_2 + a_{34}x_3); \end{array} \quad 5): \begin{array}{l} x_1 = x_1; \\ x_2 = x_2; \\ x_3 = x_3; \\ x_4 = x_4. \end{array} \quad (7)$$

The composition of these operators looks like this:

$$1)\circ 5): \begin{array}{l} x_1 = x_1(1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4); \\ x_2 = x_2(1 + a_{12}x_1 - a_{23}x_3 - a_{24}x_4); \\ x_3 = x_3(1 + a_{13}x_1 + a_{23}x_2 - a_{34}x_4); \\ x_4 = x_4(1 + a_{14}x_1 + a_{24}x_2 + a_{34}x_3); \end{array} \quad (8)$$

Lemma 2. The following statements hold for the $1)\circ 5)$ operators:

i) there are no fixed points other than the simplex vertices *I*, *P*, *H*, and *D*;

ii) the $1)\circ 5)$ operator *I* vertice is a repeller, the vertices *P* and *H* are saddles, and the vertice *D* is an attractor;

II)

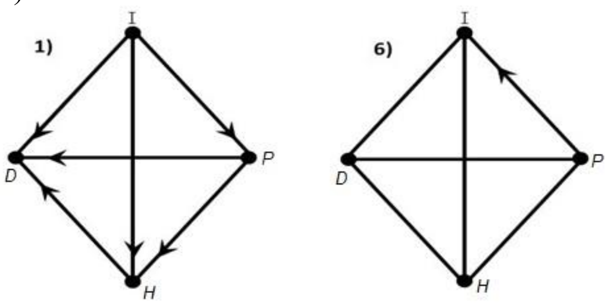

Figure 4. *A fully directed tournament and a graph with one edge directed.*

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$$1): \begin{cases} x_1 = x_1(1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4); \\ x_2 = x_2(1 + a_{12}x_1 - a_{23}x_3 - a_{24}x_4); \\ x_3 = x_3(1 + a_{13}x_1 + a_{23}x_2 - a_{34}x_4); \\ x_4 = x_4(1 + a_{14}x_1 + a_{24}x_2 + a_{34}x_3); \end{cases} \quad 6): \begin{cases} x_1 = x_1(1 + b_{12}x_2); \\ x_2 = x_2(1 - b_{12}x_1); \\ x_3 = x_3; \\ x_4 = x_4. \end{cases} \qquad (9)$$

The composition of these operators looks like this:

$$1)\circ 6): \begin{cases} x_1 = x_1(1 + b_{12}x_2)(1 - a_{12}x_2(1 - b_{12}x_1) - a_{13}x_3 - a_{14}x_4); \\ x_2 = x_2(1 - b_{12}x_1)(1 + a_{12}x_1(1 + b_{12}x_2) - a_{23}x_3 - a_{24}x_4); \\ x_3 = x_3(1 + a_{13}x_1(1 + b_{12}x_2) + a_{23}x_2(1 - b_{12}x_1) - a_{34}x_4); \\ x_4 = x_4(1 + a_{14}x_1(1 + b_{12}x_2) + a_{24}x_2(1 - b_{12}x_1) + a_{34}x_3); \end{cases} \qquad (10)$$

Lemma 3. The following statements hold for the $1)\circ 6)$ operators:

i) there are fixed points on Γ_{IP} edges in addition to the simplex vertices *I*, *P*, *H*, and *D*;

ii) the vertices *I*, *P* and *H* of the $1)\circ 6)$ operators are saddles and the vertex D is an attractor point;

iii) the fixed point of the $1)\circ 6)$ operator on Γ_{IP} edges is a saddle point.

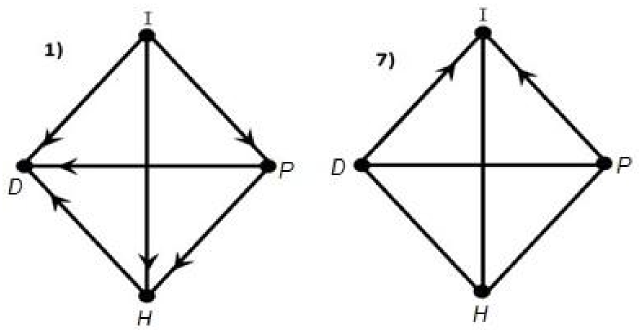

Figure 5. *A fully directed tournament and a graph with two edges directed*

$$1): \begin{cases} x_1 = x_1(1 - a_{12}x_2 - a_{13}x_3 - a_{14}x_4); \\ x_2 = x_2(1 + a_{12}x_1 - a_{23}x_3 - a_{24}x_4); \\ x_3 = x_3(1 + a_{13}x_1 + a_{23}x_2 - a_{34}x_4); \\ x_4 = x_4(1 + a_{14}x_1 + a_{24}x_2 + a_{34}x_3); \end{cases} \quad 7): \begin{cases} x_1 = x_1(1 + b_{12}x_2 + b_{14}x_4); \\ x_2 = x_2(1 - b_{12}x_1); \\ x_3 = x_3; \\ x_4 = x_4(1 - b_{14}x_1). \end{cases} \qquad (11)$$

The composition of these operators looks like this:

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$$1)\circ 7): \begin{cases} x_1' = x_1\left(1+b_{12}x_2+b_{14}x_4\right)\left(1-a_{12}x_2\left(1-b_{12}x_1\right)-a_{13}x_3-a_{14}x_4\left(1-b_{14}x_1\right)\right); \\ x_2' = x_2\left(1-b_{12}x_1\right)\left(1+a_{12}x_1\left(1+b_{12}x_2+b_{14}x_4\right)-a_{23}x_3-a_{24}x_4\left(1-b_{14}x_1\right)\right); \\ x_3' = x_3\left(1+a_{13}x_1\left(1+b_{12}x_2+b_{14}x_4\right)+a_{23}x_2\left(1-b_{12}x_1\right)-a_{34}x_4\left(1-b_{14}x_1\right)\right); \\ x_4' = x_4\left(1-b_{14}x_1\right)\left(1+a_{14}x_1\left(1+b_{12}x_2+b_{14}x_4\right)+a_{24}x_2\left(1-b_{12}x_1\right)+a_{34}x_3\right); \end{cases} \quad (12)$$

Lemma 4. The following statements hold for the operator $1)\circ 7)$:

i) In addition to the simplex vertices I, P, H, and D, there are fixed points on edges Γ_{IP} and Γ_{ID};

ii) The vertices I, P, and H of the operator $1)\circ 7)$ are saddles and the end D is an attractor point;

iii) The fixed points of the operator 122 on edges 45 and 56 are saddles.

References

1. Eshmamatova D. B., & Tadzhieva M. A., Ganikhodzhaev, R.N. Degenerate cases in Lotka-Volterra systems. AIP Conference Proceedings. 2781. 020034-1-8 (3, Scopus, IF=0.16) 2023
2. Ganikhodzhaev R.N.,& Eshmamatova D.B.,(2006) Quadratic automorphisms of a simplex and the asymptotic behavior of their trajectories. Vladikavkaz. Mat. Zh. (Vol.8) 12-28, 2006. Familiya I.SH. Maqola nomi, Jurnal nomi, soni, yili, sahifalari.
3. Moon J.W. Topics on Tournaments. 2013
4. Eshmamatova DB, Tadzhieva MA, Ganikhodzhaev RN. Criteria for internal fixed points existence of discrete dynamic Lotka–Volterra systems with homogeneous tournaments. Izvestiya Vysshikh Uchebnykh Zavedeniy. Prikladnaya Nelineynaya Dinamika 2022; 30 (6): 702-716.
5. Ganikhodzhaev RN, Tadzhieva MA, Eshmamatova DB. Dynamical proporties of quadratic homeomorphisms of a finite-dimensional simplex. Journal of Mathematical Sciences 2020; 245 (3): 398-402.
6. Kermack W, McKendrick AA. A contribution to the mathematical theory of epidemics. Proceedings of the Royal Society of London. Series A, Containing Papers of a Mathematical and Physical Character 1927; 115 (772): 700-721.