

RAYLEIGH WAVES IN A PARTIALLY SATURATED POROUS-ELASTIC MEDIUM

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Abstract: The propagation of Rayleigh waves in a partially saturated porous medium is investigated. The medium is modeled as a solid skeleton containing pores filled with two immiscible viscous fluids. The model also accounts for gas bubbles within the partially saturated liquid. In an isotropic medium, the potential functions yield three compressional (P) waves and one shear (S) wave, all of which decay exponentially with depth. Due to dissipative effects, the Rayleigh wave attenuates with depth, bringing the surface stresses to zero. The existence of these waves is governed by an irrational secular equation whose roots were found in polynomial form. Based on this formulation, the propagation and characteristics of the Rayleigh waves were analyzed. The wave velocity and amplitude were used to determine the mean displacement path in the medium. The existence of the waves depends on the values of the parameters in the secular equation, and the required saturation levels were identified by numerical modeling. In addition, the effects of velocity, quality factor, polarization frequency, capillary pressure, viscosity, and solid-frame anelasticity were studied.

Keywords: Surface waves; free oscillations; seismic attenuation; theoretical seismology; wave propagation.

1. Introduction

Many magmatic, metamorphic, and sedimentary rocks of the Earth's crust have a porous structure and may be saturated with one or more fluids. In such cases, the resulting partially saturated porous-elastic medium is of great importance in acoustics, civil engineering, seismology, and subsurface resource exploration. The equations developed by Biot provide the main theoretical foundation for describing wave propagation in porous-elastic media [1]. Later, researchers such as Berryman, Pride, and others extended these equations to multi-phase fluids. Based on mixture theory, the existence of additional compressional waves in partially unsaturated media was also theoretically justified [1].

Recently, complex mathematical models have been developed that account for capillary pressure, viscosity, and inertial coupling [2]. The propagation of Rayleigh waves is described by complex irrational secular equations, which must be transformed into polynomial form and solved using numerical methods[1]. Therefore, in this study, we investigate the propagation of Rayleigh waves in a partially saturated porous-elastic medium with two immiscible viscous fluids. Through modeling, the wave speed, quality factor, polarization, as well as the effects of partial saturation, fluid viscosity, and solid anelasticity are analyzed [2].

2. Basic Equations

A composite porous medium consists of three constituents: a solid frame, a pore gas, and a pore liquid, denoted by the indices s , g , and l , respectively[1]. In a porous aggregate with total connected porosity f , the volumetric fractions are defined as follows:

$$\delta_s = 1 - f, \delta_g = (1 - \sigma)f, \delta_l = \sigma f, \quad (1)$$

where θ is the saturation of the pore liquid (the fraction of liquid in the connected pore space) [2].

In an isotropic porous material with no external forces, the equations of motion for low-frequency oscillations of the constituents are given by:

$$\begin{aligned}\delta_s \tau_{ij,j}^{(p)} &= \delta_s \rho_s \ddot{u}_i - q_g (\dot{v}_i - \dot{u}_i) - q_l (\dot{w}_i - \dot{u}_i), \\ -\delta_g p_{,i}^{(g)} &= \delta_g \rho_g \ddot{v}_i + q_g (\dot{v}_i - \dot{u}_i), \\ -\delta_l p_{,i}^{(l)} &= \delta_l \rho_l \ddot{w}_i + q_l (\dot{w}_i - \dot{u}_i),\end{aligned}\quad (2)$$

Here, p_g and p_l are the pressures in the gas and liquid phases, while σ_{ij} is the stress tensor in the drained porous medium. ρ_s, ρ_g and ρ_l are the densities of the solid, gas, and liquid components. $u_i^{(s)}, u_i^{(g)}$ and $u_i^{(l)}$ denote the displacement components of the solid, gas, and liquid particles, respectively[4]. Repeated indices imply summation. A dot over a quantity indicates differentiation with respect to time, whereas a comma denotes a partial derivative with respect to a spatial coordinate[3].

The dissipation coefficients for gas (b_g) and liquid (b_l) are defined by:

$$q_k = \eta_k \delta_k^2 / (\chi \chi_k), (k = g, l), \quad (3)$$

where η_g and η_l denote the viscosities of the gas and liquid, and K is the intrinsic permeability of the porous medium[3].

The stresses in the porous medium and the pressures in the pore phases are given by:

$$\delta_s \tau_{ij}^{(p)} = (a_{11} \quad u + a_{12} \quad v + a_{13} \quad w) \delta_{ij} + G_p (u_{i,j} + u_{j,i} - \frac{2}{3} u_{k,k} \delta_{ij}), \quad (4)$$

$$-\delta_g p^{(g)} = (a_{21} \quad u + a_{22} \quad v + a_{23} \quad w), \quad (5)$$

$$-\delta_l p^{(l)} = (a_{31} \quad u + a_{32} \quad v + a_{33} \quad w), \quad (6)$$

Here, δ_{ij} is the Kronecker delta. The coefficients C_{ijkl} are the elastic constants of the medium. In terms of measurable parameters, these coefficients are defined as:

$$\delta_s \rho_s \ddot{u} = [(a_{11} + \frac{1}{3} G_p) \quad u + a_{12} \quad v + a_{13} \quad w] + G_p \quad^2 u + q_g (\dot{v} - \dot{u}) + q_l (\dot{w} - \dot{u}), \quad (7)$$

$$\delta_g \rho_g \ddot{v} = (a_{21} \quad u + a_{22} \quad v + a_{23} \quad w) - q_g (\dot{v} - \dot{u}), \quad (8)$$

$$\delta_l \rho_l \ddot{w} = (a_{31} \quad u + a_{32} \quad v + a_{33} \quad w) - q_l (\dot{w} - \dot{u}). \quad (9)$$

where K_s , K_g and K_l are the bulk moduli of the solid, gas, and liquid phases, respectively; μ is the frame bulk modulus; ζ is the degree of saturation of the solid grains (with $\zeta = 1$ corresponding to full saturation); and λ is an equivalent bulk modulus corresponding to the macroscopic capillary pressure between the wetting and non-wetting fluids[4].

3. General Solution: Harmonic Waves

In a homogeneous isotropic medium, the three displacement fields can be decomposed using Helmholtz potentials. The displacement vectors in the solid and fluids are written as:

$$u = \phi + S, \quad S = 0; \quad (10)$$

$$v = \psi + G, \quad G = 0; \quad (11)$$

$$w = \xi + L, \quad L = 0. \quad (12)$$

Here, Φ_i and Ψ_i are the dilatational (scalar) and solenoidal (vector) potentials, respectively[4].

Substituting the potentials into the governing equations leads to the following system:

$$\delta_s \rho_s \ddot{\phi} = [\tilde{a}_{11} \quad^2 \phi + a_{12} \quad^2 \psi + a_{13} \quad^2 \xi + q_g (\dot{\psi} - \dot{\phi}) + q_l (\dot{\xi} - \dot{\phi})], \quad \tilde{a}_{11} = a_{11} + \frac{4}{3} G_p, \quad (13)$$

$$\delta_g \rho_g \ddot{\psi} = a_{12}^2 \phi + a_{22}^2 \psi + a_{23}^2 \xi + q_g (\dot{\phi} - \dot{\psi}), \quad (14)$$

$$\delta_l \rho_l \ddot{\psi} = a_{13}^2 \phi + a_{23}^2 \psi + a_{33}^2 \xi + q_g (\dot{\phi} - \dot{\xi}), \quad (15)$$

$$\delta_s \rho_s \ddot{S} = G_p^2 S + q_g (\dot{G} - \dot{S}) + q_l (\dot{L} - \dot{S}), \quad (16)$$

$$\delta_g \rho_g \ddot{G} = q_g (\dot{S} - \dot{G}), \delta_l \rho_l \ddot{L} = q_l (\dot{S} - \dot{L}). \quad (17)$$

If the potentials vary harmonically in time as $e^{-i\omega t}$, then Φ_i represent monochromatic waves with angular frequency ω . In that case, Eqs. (13)–(15) become:

$$(\tilde{a}_{11}^2 + \omega^2 \Lambda_s) \phi + (a_{12}^2 - \omega^2 r_g) \psi + (a_{13}^2 - \omega^2 r_l) \xi = 0, \quad (18)$$

$$(a_{12}^2 - \omega^2 r_g) \phi + (a_{22}^2 + \omega^2 \Lambda_g) \psi + (a_{23}^2) \xi = 0, \quad (19)$$

$$(a_{13}^2 - \omega^2 r_l) \phi + (a_{23}^2) \psi + (a_{33}^2 + \omega^2 \Lambda_l) \xi = 0, \quad (20)$$

where: $\alpha^2 = \frac{\rho_a \omega^2}{G}$, $\beta^2 = \frac{\rho_b \omega^2}{M}$, $\gamma^2 = \frac{\rho_c \omega^2}{N}$, and ρ_a , ρ_b , ρ_c , G , M , N are effective density and modulus parameters of the three phases[5]. From Eqs. (19)–(20), one obtains the relations:

$$(A_1^4 + \omega^2 B_1^2 + \omega^4 C_1) \psi = (A_2^4 + \omega^2 B_2^2 + \omega^4 C_2) \phi, \quad (21)$$

$$(A_1^4 + \omega^2 B_1^2 + \omega^4 C_1) \xi = (A_3^4 + \omega^2 B_3^2 + \omega^4 C_3) \phi, \quad (22)$$

Applying these relations to Eq. (18) yields:

$$[A^6 + \omega^2 B^4 + \omega^4 C^2 + \omega^6 D] \phi = 0. \quad (23)$$

where $\zeta = \frac{K_s + \dots}{\dots}$, $\xi = \frac{\mu_s + \dots}{\dots}$, $\psi = \frac{G_s + \dots}{\dots}$. Equation (23) separates into three Helmholtz equations:

$$(\alpha^2 + \frac{\omega^2}{\alpha_i^2}) \phi_i = 0, \quad (i=1,2,3). \quad (24)$$

This result indicates the existence of three compressional (P) waves; they propagate with phase velocities v_{p1} , v_{p2} , and v_{p3} , and are described by the scalar potentials Φ_1 , Φ_2 , Φ_3 [6]. The P-wave velocities are obtained as the roots of the cubic equation:

$$D\alpha^6 - C\alpha^4 + B\alpha^2 - A = 0. \quad (25)$$

For a linear poroelastic medium, the total dilatation potential is given by:

$$\phi = \phi_1 + \phi_2 + \phi_3, \quad (26)$$

From (21)–(22) one then obtains:

$$\psi = \mu_1 \phi_1 + \mu_2 \phi_2 + \mu_3 \phi_3, \quad (27)$$

$$\xi = \nu_1 \phi_1 + \nu_2 \phi_2 + \nu_3 \phi_3, \quad (28)$$

In analogy to the scalar potentials, the vector potentials $\mathbf{A}$ for harmonic shear waves satisfy:

$$(G_p^2 + \omega^2 \Lambda_s) S - \omega^2 r_g G - \omega^2 r_l L = 0, \quad (29)$$

$$\{G, L\} = \{\mu_4, \nu_4\} S, \quad \mu_4 = \frac{r_g}{\Lambda_g}, \quad \nu_4 = \frac{r_l}{\Lambda_l}. \quad (30)$$

From these relations one obtains the Helmholtz equation for the shear (SV) wave:

$$(\alpha^2 + \frac{\omega^2}{\beta^2}) S = 0, \quad \beta^2 = \frac{G_p}{\Lambda_s - \mu_4 r_g - \nu_4 r_l}, \quad (31)$$

indicating the existence of a shear wave propagating with speed $v_s = \omega / k_s$ [6].

4. Plane Motion

Consider a Cartesian coordinate system in which the half-space $x_3 \geq 0$ is occupied by the porous solid, bounded by the plane $x_3 = 0$ (the free surface). Due to isotropy, the wave motion can be studied in the $x_1 - x_3$ plane, and all fields depend only on x_1 and x_3 . The displacements of the solid and fluid phases in this plane are expressed by:

$$u_x = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial x} + \mu_4 \frac{\partial \phi_4}{\partial z}, \quad u_z = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial z} - \mu_4 \frac{\partial \phi_4}{\partial x}, \quad (32)$$

$$v_x = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial x} + \mu_4 \frac{\partial \phi_4}{\partial z}, \quad v_z = \sum_{j=1}^3 \mu_j \frac{\partial \phi_j}{\partial z} - \mu_4 \frac{\partial \phi_4}{\partial x}, \quad (33)$$

$$w_x = \sum_{j=1}^3 v_j \frac{\partial \phi_j}{\partial x} + v_4 \frac{\partial \phi_4}{\partial z}, \quad w_z = \sum_{j=1}^3 v_j \frac{\partial \phi_j}{\partial z} - v_4 \frac{\partial \phi_4}{\partial x}, \quad (34)$$

Here Φ_p and Ψ_s are the potentials of the P-wave and SV-wave, respectively, propagating with phase velocities v_p and v_s . Substituting these forms into Eqs. (4)–(6) allows the calculation of the stress components and pore pressures in terms of the potentials.

The relevant boundary conditions at the surface ($x_3 = 0$) are those of a free surface. Specifically, the resultant normal and tangential stresses must vanish, and the pressures in each pore fluid phase must be zero at the surface (open pore condition). Thus the boundary conditions are: $\sigma_{33} = 0$, $\sigma_{31} = 0$, $p_l = 0$, $p_g = 0$ at $x_3 = 0$ [7]. These conditions can be written as:

$$a) \delta_s \tau_{zz}^{(s)} - \delta_g p^{(g)} - \delta_l p^{(l)} = 0 \quad b) \tau_{zx}^{(s)} = 0 \quad c) p^{(g)} = 0 \quad d) p^{(l)} = 0 \quad (35)$$

Under these conditions, substituting the displacements $\phi_j = A_j e^{ik(x-ct)-kd_j z}$, ($j=1,2,3,4$), into the stress and pressure expressions yields four linear homogeneous equations. A nontrivial solution exists only when the determinant of the coefficient matrix is zero; this condition guarantees the existence of a Rayleigh wave.

For a fully saturated porous medium, the existence of a Rayleigh wave is governed by the following secular equation:

$$4d_4 \sum_{j=1}^3 D_j d_j (2-h) - 2 \sum_{j=1}^3 D_j - h \sum_{j=1}^3 D_j X_j = h \frac{c^2}{\beta^2}, \quad (36)$$

where

$$\alpha^2 = 2 - \frac{\rho_s}{\rho}, \quad \beta^2 = 1 - \frac{\rho_s}{\rho}, \quad \chi = \frac{\rho_s}{\rho}, \quad \zeta = \frac{Q}{N} \quad \text{and} \quad \rho = \rho_s + (1-f)\rho_l + f\rho_g, \quad G \quad \text{and} \quad N \quad \text{are}$$

effective moduli of the drained frame [7]. If $\zeta = 1$ and $K_l = K_g$, Eq. (36) reduces to the classical secular equation for a Rayleigh wave in an elastic solid. When real roots exist, the secular equation is irrational and must be converted into polynomial form. By eliminating the square roots (through successive squaring and algebraic simplifications), Eq. (36) becomes a 16th-degree polynomial:

$$\sum_{j=0}^{16} g_j h_j = 0, \quad (37)$$

The coefficients a_n of this polynomial can be obtained using computer algebra software (e.g., MATLAB) via the relation:

$$(\Gamma_1 - \Delta_1)^4 - 2(\Gamma_1 - \Delta_1)^2 (\Delta_2 d_1^2 d_2^2 + \Gamma_2 d_3^2 d_4^2) + \Delta_4^2 d_1^4 d_2^4 + \Gamma_4^2 d_3^4 d_4^4 = 2\Delta_2 \Gamma_2 d_1^2 d_2^2 d_3^2 d_4^2, \quad (38)$$

where K_i denotes the wavenumbers satisfying the secular condition. The polynomial equation (37) has 16 complex roots, but some are spurious (do not satisfy the original Eq. (36)) and are discarded.

5. Rayleigh Waves

5.1. Non-Uniform Wave Propagation

If the pore fluids have no viscosity (no dissipation), the medium is conservative and the true surface wave travels slower than any body wave. In a multiphase medium (especially when one of the pore fluids is air or gas), the third compressional wave can have a very low velocity, causing the surface wave velocity to drop to as low as 0.1–0.9 times the shear wave velocity. Lo introduced the concept of pseudo-surface waves under these conditions, but this leads to complex secular equations whose solutions are defined only via complex roots. In this study, we use complex roots to demonstrate the existence of attenuating Rayleigh waves on the surface of a dissipative porous solid.

The phase velocity and attenuation coefficient of the surface wave are determined from the complex root k of the secular equation. In a dissipative medium, the true surface waves decay rapidly, propagating along the interface with maximum attenuation normal to the surface. If the secular equation yields only complex roots, it indicates that true surface waves may not propagate in the dissipative medium.

A porous-elastic medium saturated with a viscous fluid is an important model for sedimentary and reservoir rocks, where viscous losses in pores are significant. In this study, the elastic effect of the frame is modeled by letting the bulk modulus K_s and shear modulus G_s of the solid become complex: $K_s = K_s(1 + i/Q_K)$ and $G_s = G_s(1 + i/Q_G)$, where the quality factors Q_K and Q_G are related to the logarithmic decrement parameters of the drained matrix [8]. The Rayleigh-wave quality factor Q_R is then determined by:

$$Q^{-1} = \frac{h_R - h_l Q_s}{h_R Q_s + h_l}, \quad Q_s^{-1} = -\frac{(\beta^2)}{(\beta^2)} \quad (39)$$

when a solution of the secular equation has been found [9].

5.2. Particle Motion

Expressions (32)–(34) define the motion of solid and fluid particles in the $x-z$ plane. From these one can compute the mean displacement components of the porous medium:

$$U = \delta_s u_x + \delta_g v_x + \delta_l w_x = \sum_{j=1}^3 [(\delta_s + \delta_g \mu_j + \delta_l \nu_j) \frac{\partial \phi_j}{\partial x}] + (\delta_s + \delta_g \mu_4 + \delta_l \nu_4) \frac{\partial \phi_4}{\partial z}, \quad (40)$$

$$W = \delta_s u_z + \delta_g v_z + \delta_l w_z = \sum_{j=1}^3 [(\delta_s + \delta_g \mu_j + \delta_l \nu_j) \frac{\partial \phi_j}{\partial z}] - (\delta_s + \delta_g \mu_4 + \delta_l \nu_4) \frac{\partial \phi_4}{\partial x}, \quad (41)$$

Using the potentials from (35), the displacements are:

$$\phi_j = A_j \gamma_j e^{i(kx - \omega t) - kd_j z}, \quad (j = 1, 2, 3, 4), \quad (42)$$

where α_i ($i = 1, 2$) are the roots obtained from the singular system of boundary conditions (36).

Combining (32)–(34), the mean displacement can be written as:

$$U = i \ddot{E} U_0 e^{i \arg U_0} e^{i \omega (\frac{x}{c} - t)}, \quad W = -i \ddot{W}_0 e^{i \arg W_0} e^{i \omega (\frac{x}{c} - t)}, \quad (43)$$

where

$$\alpha^2 = k_r^2 - k_i^2, \quad \alpha_i = 2k_r k_i,$$

and $k = k_r + ik_i$ is the complex wavenumber of the Rayleigh wave [9][10]. Taking the real part of (43) yields the apparent displacement of the fluid phase:

$$u = -\tilde{E} U_0 \tilde{D} e^{-k_i x} \sin(\arg U_0 + \theta), \quad w = -\tilde{E} W_0 \tilde{D} e^{-k_i x} \cos(\arg W_0 + \theta). \quad (44)$$

Here ζ is treated as a varying parameter, and it determines the shape of the particle motion trajectory at depth z (in the $x-z$ plane). In other words, the system of Eqs. (44) parametrically describes the trajectory of the particles in the dissipative porous solid as the Rayleigh wave propagates.

6. Numerical Solution

The physical model of the porous medium is taken as a sandstone saturated with water and air. Various realistic parameter values are chosen for the water-saturated sandstone. The sandstone is assumed to have a solid-frame bulk modulus K_s , shear modulus G_s , and density ρ_s , with porosity f . The pore gas is taken with bulk modulus K_g and density ρ_g , and the pore water with bulk modulus K_l and density ρ_l . The viscous dissipation in the pores is characterized by the dissipation coefficients b_g and b_l , defined by $b_g = \mu_g / k_0$ and $b_l = \mu_l / k_0$, where μ_g , μ_l are the fluid viscosities and k_0 is a reference permeability. The wave frequency is limited to about 1 kHz to ensure low-frequency conditions [10].

Typical logarithmic decrement values δ for elastic materials have been estimated by Stoll (1985) [11]. These values are used in (41) to compute the quality factors for the frame moduli. In the chosen numerical model, the existence of a Rayleigh wave depends on the saturation ζ of the connected pore space. In other words, the Rayleigh wave in the model exists only for certain ranges of ζ [11]. A portion of the numerical study is devoted to determining the range of saturation for which the Rayleigh wave exists. The remainder investigates the particle motion in the corresponding saturated medium (typically $\zeta = 1$).

The existing Rayleigh wave is represented by the complex velocity obtained from the root k . The velocity ratio c_R / v_s and the shear-wave velocity v_s are determined from k . The quality factor Q_R of the Rayleigh wave is computed from the complex P- and S-wave speeds using Eq. (42).

Next, the effects of various model parameters on the Rayleigh wave are studied, focusing on both its existence range (in terms of ζ) and its polarization for given ζ . These parameters include wave frequency, capillary pressure P_c , pore-fluid viscosity, and frame anelasticity (logarithmic decrement). It is observed that, in the chosen model, variations of these parameters have negligible effect on the shear-wave velocity v_s and its quality factor Q_s [11]. The dimensionless real quantities c_R / v_s and Q_R are key measures for the Rayleigh wave. Note that c_R / v_s allows one to compute the complex Rayleigh wave velocity $v_R = v_s (c_R / v_s)$. The values of c_R / v_s for the existing Rayleigh waves are presented in Figures 1–4. Detailed discussion is given in the following figure captions.

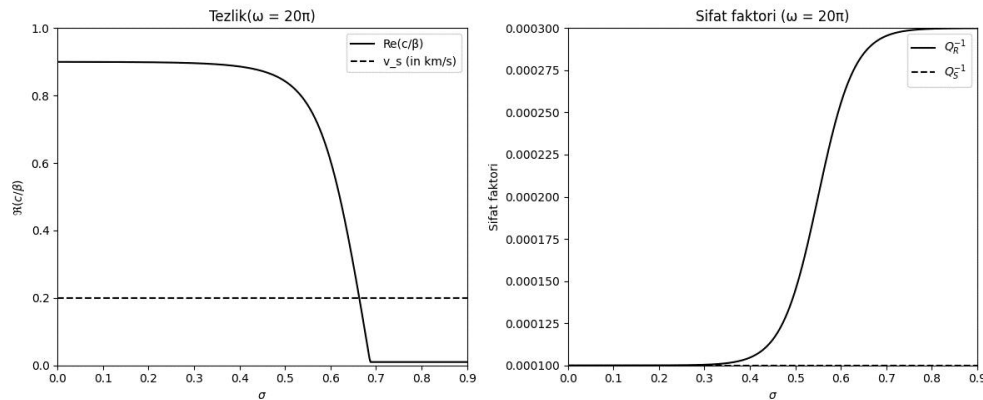


Figure 1. Variation of the velocity ratio c_R / v_s and quality factor Q_R as functions of saturation ζ and frequency ω . The porous elastic medium is saturated with a viscous liquid.

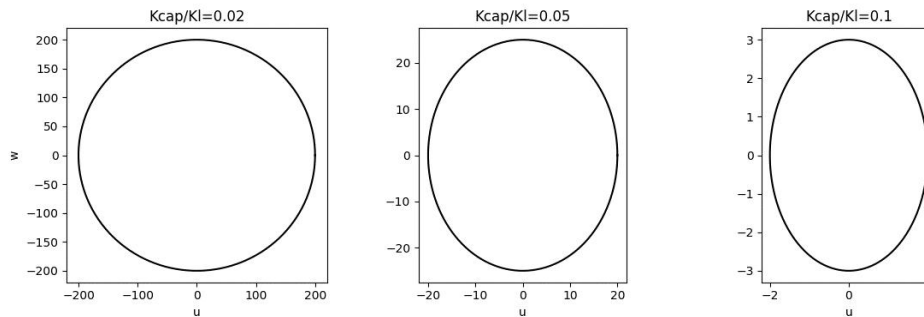


Figure 2. Dependence of particle motion on depth ζ and capillary pressure P_c . The porous elastic medium is saturated with $\zeta = 1$ of a viscous liquid.

7. Conclusion

In this work, the exponential variation of the Rayleigh wave in porous media saturated with a mixture of two immiscible liquids has been studied. When one of the pore fluids is air or rarefied gas, the model represents a partially saturated porous medium. The considered medium is not lossless: energy dissipation arises from losses in the solid frame and viscous dissipation in the pores. In the presence of dissipation, the Rayleigh wave's existence and propagation are determined by an irrational secular equation. This equation is transformed into polynomial form, and its roots are found numerically. The roots are selected based on the condition that they ensure attenuation of the displacements with depth. The resulting roots determine the complex velocity of the Rayleigh wave in a partially saturated dissipative porous solid[12].

A numerical example was conducted to illustrate the method. Based on calculations and analysis for a dissipative porous solid, the following useful conclusions were drawn[12]:

- The Rayleigh wave does not exist for any value of ζ , but only within a certain segment of ζ . In the dry porous frame limit, the wave speed approaches zero at the upper bound of this range, and a small increase in ζ leads to a steep drop in speed.
- In the saturated porous-elastic medium, the wave speed does not drop to zero; instead it attains a minimum before reaching its maximum, and this behavior is accompanied by strong attenuation.
- As saturation increases, the particle motion decays more rapidly with depth. The elliptical trajectory is preserved both at the surface and below, while the attenuation increases with ζ .
- The quality factor of the Rayleigh wave is proportional to frequency and fluid viscosity, whereas the particle motion is almost independent of these.

- When capillary pressure is low, a higher saturation is required for the Rayleigh wave to exist. As the pressure increases, the trajectory ellipticity decreases and the particle motion becomes more vertical with depth.

A generalized model thus obtained can represent conditions from nearly dry to fully saturated by varying the fluid saturation. Since seismic sources and receivers are often near the surface, analyzing Rayleigh waves as a function of saturation is important for exploration seismology and understanding pre-earthquake processes[12].

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