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ADVANCED APPROACHES TO SUMMATION IN MATHEMATICAL EDUCATION

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Abstract: This article explores the theoretical and practical aspects of efficiently and optimally calculating certain types of mathematical sums that are widely used in mathematical analysis and algebraic structures. Since secondary school mathematics textbooks typically address only basic summation cases using arithmetic or geometric progression formulas, students are often limited to solving simple summation problems. As a result, they face difficulties when encountering more complex expressions. In this study, general formulas for various types of sums, their mathematical proofs, conditions of applicability, and illustrative examples are thoroughly discussed. In addition, several problems related to identities and summations frequently encountered in contemporary mathematical olympiads and academic competitions are analyzed. This work serves as a theoretical and methodological resource for teaching mathematics, developing computational algorithms, and enhancing students' mathematical thinking.[1-5].

Keyword:Mathematics education,Sums,Sum calculation methods, Advanced approache, Innovative teaching, Algebraic sums.

1-M. Calculate the sum: $S = a + aa + aaa + \ldots + \underbrace{aaa\ldots a}_{n \text{ ta}}$

Solution. To evaluate this sum, we transform the given expression into the following form:

$$S = a + aa + aaa + \ldots + \underbrace{aaa\ldots a}_{n \text{ ta}} = \frac{a}{9}\left(9 + 99 + 999 + \ldots + \underbrace{999\ldots 9}_{n \text{ ta}}\right) =$$

$$= \frac{a}{9}\left(10 - 1 + 10^2 - 1 + 10^3 - 1 + \ldots + 10^n - 1\right) = \frac{a}{9}\left(10 + 10^2 + 10^3 + \ldots + 10^n - n\right) =$$

$$= \frac{a}{9}\left(\frac{10\left(10^n - 1\right)}{10 - 1} - n\right) = \frac{a}{9}\left(\frac{10\left(10^n - 1\right)}{9} - n\right) = \frac{a\left(10^{n+1} - 10 - 9n\right)}{81}$$

As a result, $S = a + aa + aaa + \ldots + \underbrace{aaa\ldots a}_{n \text{ ta}}$ we find $S = \frac{a\left(10^{n+1} - 10 - 9n\right)}{81}$ that the given sum can be evaluated using a known.[5-9].

2-M. $\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \ldots + \frac{1}{1+2+3+4+\ldots+n}$ evaluate the sum.

Solution. It is evident that the denominators form the sum of the first nnn terms of an arithmetic progression. Therefore,

$$\frac{1}{1+2} + \frac{1}{1+2+3} + \frac{1}{1+2+3+4} + \ldots + \frac{1}{1+2+3+4+\ldots+n} = \frac{1}{\frac{1+2}{2}\cdot 2} + \frac{1}{\frac{1+3}{2}\cdot 3} + \frac{1}{\frac{1+4}{2}\cdot 4} +$$

$$+ \ldots + \frac{1}{\frac{1+n}{2}\cdot n} = \frac{2}{2\cdot 3} + \frac{2}{3\cdot 4} + \frac{2}{4\cdot 5} + \ldots + \frac{2}{n(n+1)} = 2\left(\frac{1}{2} - \frac{1}{3} + \frac{1}{3} - \frac{1}{4} + \frac{1}{4} - \frac{1}{5} + \ldots + \frac{1}{n} - \frac{1}{n+1}\right) =$$

$$= 2\left(\frac{1}{2} - \frac{1}{n+1}\right) = \frac{n+1-2}{n+1} = \frac{n-1}{n+1}$$

We obtain the result. Thus, the value of the given sum $\frac{n-1}{n+1}$ i s equal.[10-13]

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3-M. Calculate: $S = \frac{1}{3^2+1} + \frac{1}{4^2+2} + \frac{1}{5^2+3} + \ldots.$

Solution. First, we write down the general term formula and then perform the corresponding calculations $a_n = \frac{1}{(n+2)^2+n} = \frac{1}{n^2+5n+4} = \frac{1}{3}\left(\frac{1}{n+1} - \frac{1}{n+4}\right)$ We find its value. Now, we apply this formula to the given sum

$$S = \frac{1}{3^2+1} + \frac{1}{4^2+2} + \frac{1}{5^2+3} + \ldots. = \frac{1}{3}\left(\frac{1}{2} - \frac{1}{5} + \frac{1}{3} - \frac{1}{6} + \frac{1}{4} - \frac{1}{7} + \frac{1}{5} - \frac{1}{8} + \frac{1}{6} - \frac{1}{9} + \ldots.\right) =$$

$$= \frac{1}{3}\left(\frac{1}{2} + \frac{1}{3} + \frac{1}{4}\right) = \frac{13}{36}$$

4-M. Calculate the sum: $S = \frac{1}{1\cdot 2\cdot 3} + \frac{1}{2\cdot 3\cdot 4} + \frac{1}{3\cdot 4\cdot 5} + \ldots + \frac{1}{98\cdot 99\cdot 100}$

Solution. We write down the general term formula and then perform the corresponding calculations,

$$a_n = \frac{1}{n(n+1)(n+2)} = \frac{1}{2}\cdot\frac{2}{n(n+1)(n+2)} = \frac{1}{2}\cdot\frac{(n+2)-n}{n(n+1)(n+2)} = \frac{1}{2}\left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right)$$

$$a_n = \frac{1}{2}\left(\frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)}\right) = \frac{1}{2}\left(\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right)$$

We find its value. Now, we apply this formula to the given sum,

$\sum_{k=1}^{n} a_k = \sum_{k=1}^{n} \frac{1}{2}\left(\left(\frac{1}{n} - \frac{1}{n+1}\right) - \left(\frac{1}{n+1} - \frac{1}{n+2}\right)\right) = \frac{1}{2}\left(\left(\frac{1}{1} - \frac{1}{99}\right) - \left(\frac{1}{2} - \frac{1}{100}\right)\right) = \frac{1}{2}\left(\frac{1}{2} - \frac{1}{99\cdot 100}\right)$.[13-18].

5-M. Calculate the sum: $S = \frac{1}{5} + \frac{2}{5^2} + \frac{3}{5^3} + \ldots + \frac{n}{5^n} + \ldots$

Solution. We multiply both parts of the given sum by 5. Then,

$5S = 1 + \frac{2}{5} + \frac{3}{5^2} + ... + \frac{n}{5^{n-1}} + ...$ We form an equation. Subtracting the given equation from

$$4S = 1 + \left(\frac{2}{5} - \frac{1}{5}\right) + \left(\frac{3}{5^2} - \frac{2}{5^2}\right) + ... + \left(\frac{n}{5^{n-1}} - \frac{n-1}{5^{n-1}}\right) + ... =$$

this $= 1 + \frac{1}{5} + \frac{1}{5^2} + ... + \frac{1}{5^{n-1}} + ... = \frac{1}{1 - \frac{1}{5}} = \frac{5}{4}$

6-M. Calculate the sum: $S = x + 2x^2 + 3x^3 + 4x^4 + ...$, in this, $|x| < 1$.

Solution. We divide the $S = x + 2x^2 + 3x^3 + 4x^4 + ...$ given sum by x. As a result

Consequently $\frac{S}{x} = 1 + 2x + 3x^2 + 4x^3 + ...$ An equation is formed. Now, $\frac{S}{x} - S = 1 + x + x^2 + x^3 + ...$ We examine the difference. It is evident that this corresponds to the sum of an infinitely decreasing geometric progression. Therefore, $\frac{S}{x} - S = \frac{1}{1-x}$ It holds. Simplifying the final equation, $S = \frac{x}{(1-x)^2}$ we obtain the result.[18-21].

7-M. Calculate the sum: $\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + ... + \frac{999}{1000!}$

Solution. If we express the given sum in the following form and perform the necessary simplifications:

$$\frac{1}{2!} + \frac{2}{3!} + \frac{3}{4!} + ... + \frac{999}{1000!} = \frac{2-1}{2!} + \frac{3-1}{3!} + \frac{4-1}{4!} + ... + \frac{1000-1}{1000!} = \frac{1}{1!} - \frac{1}{2!} + \frac{1}{2!} - \frac{1}{3!} + \frac{1}{3!} - \frac{1}{4!} +$$

$$+ ..... + \frac{1}{999!} - \frac{1}{1000!} = 1 - \frac{1}{1000!}$$

we obtain the result.

8-M. $S = 1 + 2x + 3x^2 + 4x^3 + ... + nx^{n-1}$ calculate the sum.

Solution. **1-usul.** To compute the given sum conveniently, we first consider it as a function and find its (denoted as Q) antiderivative. Then, $Q = x + x^2 + x^3 + ... + x^n = \frac{x(x^n - 1)}{x - 1}$ It works. Here, we used the formula for the sum of the first n terms of a geometric progression. Now, if we differentiate the final formula we obtained, we get the value of the sum

s $S = Q' = \left(\frac{x^{n+1} - x}{x - 1}\right)' = \frac{((n+1)x^n - 1)(x-1) - (x^{n+1} - x)}{(x-1)^2} = \frac{(nx - n - 1)x^n + 1}{(x-1)^2}$.

That is to say $S = 1 + 2x + 3x^2 + 4x^3 + ... + nx^{n-1} = \frac{(nx - n - 1)x^n + 1}{(x-1)^2}$ will be.

2-usul. If we multiply both parts of the given sum by xxx, then $Sx = x + 2x^2 + 3x^3 + 4x^4 + ... + nx^n$ We get the result. If we subtract this sum from the given equation, $S - Sx = 1 + x + x^2 + x^3 + ... + x^{n-1} - nx^n = \frac{x^n - 1}{x - 1} - nx^n$ It works. From this, then $-S(x-1)=\frac{x^n-1}{x-1}-nx^n$ $S=\frac{nx^n(x-1)-x^n+1}{(x-1)^2}=\frac{(nx-n-1)x^n+1}{(x-1)^2}$ results in.[22-25].

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