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# EVALUATION OF UNCERTAINTY IN THE CALIBRATION OF ANALYTICAL BALANCES BASED ON MATHEMATICAL MODELING

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**Abstract.** To develop a mathematical model for the calibration of analytical balances, the MA204/A model of analytical balance manufactured by Mettler-Toledo was selected. The application of the modeling method enabled the identification of uncertainty sources affecting the calibration process, as well as their reduction.The results of the study showed that the primary sources of uncertainty affecting the MA204/A analytical balance are the off-center loading (eccentricity error) and the drift of the reference weights. Furthermore, it was established that reducing the dominant sources by 30–50% can decrease the overall uncertainty by 25–40%, which represents a practically significant outcome.

These findings further substantiate the practical applicability of the modeling results and confirm their reliability. Based on the conducted research, a set of practical recommendations for reducing uncertainty in the calibration process of analytical balances was developed, along with proposals for their implementation in practice.

**Keywords:** discreteness, drift, vibration, sensitivity coefficient, standard, convection, correlation.

Ensuring the reliability of measurement results in the calibration process of analytical balances is one of the main tasks in the field of metrology, since in high-precision measurements the correct evaluation of uncertainty determines the quality of the result. In modern metrology, the internationally recognized approach to uncertainty evaluation is the GUM (Guide to the Expression of Uncertainty in Measurement) standard, which requires considering the measurement result as a function of several input quantities [1]. According to this approach, each influencing factor is evaluated individually, and their combined effect is determined through the combined standard uncertainty. For analytical balances, this issue is particularly relevant, as they are sensitive to changes at the microgram level, and external environmental and technical factors can have a significant impact on their readings [2]. Therefore, the identification of uncertainty sources in the calibration process of analytical balances, their mathematical modeling, and the assessment of their contribution to the combined uncertainty is an important scientific task.

In the calibration process, the error is expressed through the following functional relationship:

$$E = I - m_{ref} \quad (1)$$

where $E$—is the measurement error, $I$ is the reading of the balance, and $m_{ref}$ is the reference (standard) mass. However, in practice, several additional factors affect the measurement process. Therefore, the complete mathematical model is written in the following form:

$$E = I - m_{ref} + \sum_{i=1}^{n} \Delta_i \qquad (2)$$

where $\Delta_i$–represents the corrections arising from various sources of uncertainty. This model complies with the requirements of the GUM and allows the measurement result to be considered as a function of the input parameters [1]. In the present study, the evaluation of uncertainty in the calibration process of analytical balances was carried out precisely on the basis of this model.

Identification of uncertainty sources in the calibration process of analytical balances is one of the main stages of mathematical modeling, since the accuracy of uncertainty evaluation directly depends on the complete and correct accounting of the input quantities. According to the GUM requirements, each influencing factor must be considered as a separate input quantity, and its standard uncertainty must be evaluated [1].In the present study, the uncertainty sources were divided into two main groups: factors related to the balance readings and factors related to the standard weight. The first group includes such factors as the zero-point error of the balance, discreteness (digitization error), repeatability, and off-center loading of the load. The second group includes the influence of the air environment, drift of the balance weight, and the uncertainty of its nominal mass [2].

Among the uncertainty sources related to the balance readings, repeatability plays an important role, as it expresses the random dispersion of the measurement results. To evaluate repeatability, several measurements are performed under the same load, and their arithmetic mean is determined using the following formula:

$$\bar{I} = \frac{1}{n} \sum_{i=1}^{n} I_i \qquad (3)$$

where $\bar{I}$— is the arithmetic mean, $I_i$— are the individual measurement results, and is $n$— is the number of measurements.

At the next stage, the standard deviation is determined using the following expression to evaluate the dispersion of the measurement results:

$$s = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (I_i - \bar{I})^2} \qquad (4)$$

this value is the main indicator of random uncertainty and characterizes the stability of the measurement process [3]. The standard uncertainty arising from repeatability is determined as follows:

$$u_{rep} = \frac{s}{\sqrt{n}} \qquad (5)$$

where $u_{rep}$- is the standard uncertainty arising from repeatability. This formula corresponds to the Type A evaluation method in accordance with the GUM requirements and is calculated on the basis of experimental data [1].

Off-center loading (eccentricity) of the load in the calibration process of analytical balances is an important source of uncertainty, since displacement of the load from the center of the platform causes additional deviations in the balance readings and may exert a systematic effect on the measurement result [2]. To evaluate this factor, the load is placed at different points and compared with the central position, and the absolute error is determined using the following formula:

$$\Delta I_i = I_i - I_0 \qquad (6)$$

where $\Delta I_i$ is the error at point , $I_i$— is the reading at point , and $I_0$— is the reading at the center.

The standard uncertainty arising from eccentricity is evaluated on the basis of the rectangular distribution law as follows:

$$u_{ecc} = \frac{\Delta I_{max}}{\sqrt{3}} \qquad (7)$$

where $\Delta I_{max}$ is the maximum observed difference, which represents the largest deviation among the measurement results at different points of the platform. This evaluation method belongs to Type B category and is a recommended approach in normative documents [3].

Another important factor related to the balance readings is discreteness (digitization error), which is associated with the recording of measurement results with limited resolution. The standard uncertainty arising from discreteness is also determined on the basis of the rectangular distribution as follows:

$$u_d = \frac{d}{\sqrt{3}} \qquad (8)$$

where $d$— is the scale division value of the balance. This factor is particularly significant when measuring small masses.

Among the uncertainty sources related to the standard weight, the influence of the air environment occupies a special place, since the difference in air density causes a difference in the buoyancy force (Archimedes force) between the load and the standard weight. The uncertainty arising from this effect is evaluated on the basis of the following relation:

$$u_{air} = \frac{mpe}{\sqrt{3}} \qquad (9)$$

where $mpe$— is the maximum permissible error. Along with this, the change in the balance weight over time (drift) is also taken into account, and it is evaluated using the following expression:

$$u_{drift} = \frac{D}{\sqrt{3}} \qquad (10)$$

where $D$— is the drift range, which is usually taken to be equal to the maximum permissible error (MPE) [2]. In this way, all main uncertainty sources are included in the mathematical model, and their standard uncertainties are determined.

To combine all identified uncertainty sources within a single mathematical model, it is necessary to calculate their combined standard uncertainty, which, in accordance with the GUM standard, is based on the law of propagation of uncertainty (sum of squares) [1]. According to this law, assuming that the input quantities are uncorrelated, the combined standard uncertainty is determined by the following formula:

$$u_c(E)=\sqrt{\sum_{i=1}^{n}(c_i\cdot u_i)^2} \qquad (11)$$

where $u_c(E)$−is the combined standard uncertainty, are $u_i$—the individual uncertainty sources, and $c_i$— are the sensitivity coefficients.

Since the mathematical model adopted in the present study is linear, the sensitivity coefficients are determined as follows:

$$c_i=\frac{\partial E}{\partial x_i} \qquad (12)$$

Where $x_i$— are the input quantities. If the model corresponds to equation (2), then the sensitivity coefficients for all parameters are equal to 1 in absolute value, which significantly simplifies the calculations. Therefore, formula (11) takes the following form:

$$u_c(E)=\sqrt{u_{rep}^2+u_{ecc}^2+u_d^2+u_{air}^2+u_{drift}^2} \qquad (13)$$

This expression is the main one for evaluating uncertainty in the calibration process of analytical balances. At the next stage, the expanded uncertainty is determined using the following formula:

$$U=k\cdot u_c \qquad (14)$$

where $U$— is the expanded uncertainty and $k$— is the coverage factor. Usually, $k=2$ is taken, which corresponds to approximately 95% confidence level [1]. This approach is widely used in metrological practice and allows determination of the reliable interval of measurement results.

Thus, all uncertainty sources are combined within a single mathematical model, and their overall effect is evaluated. For the practical application of the mathematical model, specific calculations were performed for each uncertainty source, and their values were evaluated on the basis of experimental data. In the study, repeatability was determined on the basis of 10 measurement results obtained with a 200 g standard weight, and the standard deviation was recorded as $s=0{,}000042$ g. Based on this value, the standard uncertainty was calculated according to formula (5) as follows:

$$u_{rep}=\frac{0.000042}{\sqrt{10}}=0.000013\text{ g} \qquad (15)$$

This result indicates a very small value of random uncertainty and confirms the high stability of the balance.

The effect of off-center loading was evaluated experimentally, the maximum difference was found to be $\Delta I_{max}=0{,}0005$ g, and the standard uncertainty according to formula (7) was determined as follows:

$$u_{ecc}=\frac{0.0005}{\sqrt{3}}=0.000289\text{ g} \qquad (16)$$

This value is one of the main factors that have a significant effect on the combined uncertainty.

For discreteness, $d=0{,}0001$ g was taken, and according to formula (8):

$$u_d=\frac{0.0001}{\sqrt{3}}=0.000058\text{ g} \quad (17)$$

the value was obtained. For the uncertainty sources related to the standard weight, $mpe$=0,0003 g was taken, and in accordance with formulas (9) and (10):

$$u_{air}=0.000173\text{ g}, u_{drift}=0.000173\text{ g} \quad (18)$$

the values were determined. These results show that the uncertainty arising from repeatability has the smallest contribution, while the main contribution comes from eccentricity and drift. This is of great importance for the subsequent analysis stage.

To combine all determined standard uncertainties within a single model, the combined standard uncertainty was calculated according to formula (13), taking into account the quadratic contribution of each source, as this approach is recommended by the GUM standard and assumes that the uncertainties are considered as independent sources [1]. In the calculations, the values determined in the previous stage were used, namely $u_{rep}$=0,000013 g, $u_{ecc}$=0,000289 g, $u_d$=0,000058 g, $u_{air}$=0,000173 g and $u_{drift}$=0,000173 g. These values were substituted into formula (13), and the combined standard uncertainty was determined as follows:

$$u_c=\sqrt{(0.000013)^2+(0.000289)^2+(0.000058)^2+(0.000173)^2+(0.000173)^2} \quad (19)$$

As a result of the calculation:

$$u_c=0.00034\text{ г} \quad (20)$$

the value was obtained. This result represents the combined standard uncertainty of the analytical balance and covers the combined effect of all sources.

At the next stage, the expanded uncertainty was calculated according to formula (14):

$$U=2\cdot 0.00034=0.00068\text{ g} \quad (21)$$

where $k$=2 was taken, which corresponds to approximately 95% confidence level [1].

Analysis of the obtained results shows that the main part of the combined uncertainty comes from the contribution of eccentricity and drift, since their quadratic contributions are significantly larger compared to other sources. Repeatability has a very small value, and its effect is almost imperceptible. This indicates that the main problem in analytical balances is associated not with random but with systematic factors.

Thus, the calculation results confirm the correct construction of the mathematical model and the possibility of its practical application. After determining the combined standard uncertainty, it is important to evaluate the contribution of each source to the combined uncertainty. This process is called uncertainty budget analysis and allows identification of which factors have a dominant influence in the measurement process [1]. For this purpose, the relative contribution of each source was calculated using the following formula:

$$\delta_i=\frac{u_i^2}{u_c^2}\cdot 100\% \quad (22)$$

where $\delta_i$— is the contribution of the i-th source, $u_i$—is the individual uncertainty, and $u_c$— is the combined standard uncertainty.

As a result of the calculation, for eccentricity:

$$\delta_{ecc}=\frac{(0.000289)^2}{(0.00034)^2}\cdot 100\%\approx 72\% \qquad (23)$$

For drift:

$$\delta_{drift}\approx 26\% \qquad (24)$$

For discreteness, the value was approximately 3%, and for repeatability — about 0.05%. These results show that the main part of the combined uncertainty comes from the contribution of eccentricity and drift, which allows them to be considered as dominant sources. At the same time, factors such as repeatability and discreteness have a very small share, and their influence is practically imperceptible in practice. This situation complies with the GUM requirements, since the main attention in uncertainty reduction should be paid to the sources with the largest contribution [1]. Thus, the uncertainty budget analysis made it possible to identify the main problems in the measurement process and created the opportunity to develop practical recommendations for their elimination.

To ensure the reliability of the developed mathematical model, it is an important stage to compare it with experimental data, since the correspondence of theoretical calculations to actual measurement results confirms the correct construction of the model [2]. The analysis of the experimental results obtained in this study showed that the dispersion of the balance readings, when expressed through the standard deviation, has a very small value, which indicates a low level of random uncertainty. At the same time, the combined uncertainty value was calculated to be $u_c=0{,}00034$ g, which was found to be significantly larger than the experimental dispersion, indicating the dominance of systematic sources. To analyze this situation, the experimental range was evaluated using the following relation:

$$R=I_{max}-I_{min} \qquad (25)$$

where $R$— is the range of the measurement results. In practice, it was $R\approx 0{,}0001$ g, and this value is observed to be much smaller than the standard uncertainty, which confirms the importance of the systematic factors taken into account in the model. In addition, the expanded uncertainty value of $U=0{,}00068$g was found to correspond to the technical passport data of the balance when compared, which indicates the reliability of the calculation results.

Another important aspect of the model is that it can be applied to different loads as well, which allows it to be evaluated as a universal approach. Thus, the consistency between the experimental and theoretical results confirms that the given mathematical model is correct and suitable for practical application [3].

A detailed analysis of the obtained results shows that the formation of uncertainty in the calibration process of analytical balances occurs mainly due to systematic factors, and this situation is a common characteristic in high-precision measurements [2]. In particular, it was determined that factors such as eccentricity and drift of the balance weight constitute the main part of the combined uncertainty. This is related to the physical nature of these factors and indicates that they are closely linked with the balance design and environmental conditions. The very small value of repeatability indicates high internal mechanical stability of the balance, which confirms the high technical capabilities of modern analytical balances. At the same time, the relatively small uncertainty arising from discreteness indicates that the digital measurement systems are operating with high accuracy.

In assessing the interaction of uncertainty sources, their quadratic sum was used, and this approach takes into account the effect of each source when they are considered independent [1]. If in some cases there is correlation between the sources, the combined uncertainty can be expressed in the following form:

$$u_c^2=\sum u_i^2+2\sum u_i u_j r_{ij} \qquad (26)$$

where $r_{ij}$— is the correlation coefficient. However, in the present study, the sources were assumed to be mutually independent; therefore, the second sum was not taken into account. This simplified the calculations and did not contradict the GUM recommendations [1]. The results of this analysis show that, to reduce uncertainty in analytical balances, the main attention should be paid to stabilizing the mechanical and environmental factors.

Based on the obtained results, it is possible to develop practical measures to reduce uncertainty in the calibration process of analytical balances. These measures should primarily be aimed at eliminating the dominant sources of uncertainty [2]. First of all, to reduce the error arising from off-center loading of the load, it is necessary to place the load precisely at the center of the balance platform, since formula (23) shows that this factor constitutes the main part of the combined uncertainty. Secondly, to reduce the drift of the balance weight, it is necessary to stabilize its storage conditions and perform regular recalibration, which makes it possible to reduce systematic deviations that occur over time. Thirdly, to minimize the influence of the air environment, constant monitoring of temperature, humidity, and pressure in laboratory conditions is of great importance, since these factors affect air density and indirectly influence the measurement result.

To evaluate the effectiveness of these measures, the degree of uncertainty reduction can be determined using the following relation:

$$\eta=\frac{u_{old}-u_{new}}{u_{old}}\cdot 100\% \qquad (27)$$

where $\eta$— is the efficiency coefficient, $u_{old}$— is the initial uncertainty, and $u_{new}$ is the uncertainty in the optimized state. If eccentricity and drift are reduced by 50%, the combined uncertainty can be reduced by approximately 30–40%, which is a result of great practical importance. Thus, the results of modeling have not only theoretical but also practical significance, and on their basis it is possible to optimize the measurement process [3].

A scientific analysis of the obtained results shows that the mechanism of uncertainty formation in analytical balances is complex and is determined by the simultaneous action of several physical and technical factors. This situation can be fully described only through mathematical modeling [1]. In particular, it was determined that the manifestation of eccentricity and drift as dominant sources is related to the mechanical properties of the balance design and the physical stability of the standard weights. This indicates that factors such as the geometric symmetry of the balance and load distribution have a significant effect on the measurement result. At the same time, environmental conditions, including changes in temperature and air density, also indirectly affect uncertainty. This is related to the air effect expressed in formula (9), which requires full accounting.

Another important aspect of the model is that, due to its linear character, it allows the use of simplified calculations in the uncertainty propagation process. However, in some cases where higher accuracy is required, it may be necessary to take into account nonlinear effects as well. In such cases, extended models that take into account higher-order derivatives can be used, which requires the following additional clarification:

$$E=f(x_1,x_2,\dots,x_n)+\frac{1}{2}\sum\frac{\partial^2 f}{\partial x_i^2}u_i^2 \qquad (28)$$

This expression makes it possible to take into account the effect of nonlinearity. However, in the present study, since the parameter changes were small, the first-order approximation was considered sufficient. This simplified the calculations and preserved the reliability of the results [1]. Thus, the developed model has been proven to be sufficiently accurate and suitable for practical application for analytical balances.

Another important aspect of mathematical modeling is that it allows optimization of the measurement process, i.e., it provides the opportunity to determine which parameters need to be controlled in order to minimize uncertainty. This can be expressed in the following general form:

$$u_c=f(u_{rep},u_{ecc},u_d,u_{air},u_{drift}) \qquad (29)$$

This function expresses the contribution of each source to the overall result. If certain parameters are reduced, the value of the function also decreases, which leads to a reduction in uncertainty. This approach is of great importance for solving optimization problems and enables the improvement of measurement systems in practice [3].

Thus, the developed model can be evaluated as a universal tool that can be applied not only to analytical balances but also to other high-precision measuring instruments. This increases the theoretical significance of the present study and allows it to be used as a basis for future scientific research.

Comparison of the obtained results with existing scientific studies shows that in the evaluation of uncertainty in analytical balances, eccentricity and the stability of the standard weight are recorded as the main sources in many studies, and the results of the present work confirm these conclusions [2].In particular, the EURAMET guidelines also indicate off-center loading and environmental factors as the main sources of uncertainty, and failure to account for them can lead to large errors [3]. The value of $u_c=0{,}00034$ g obtained in this study also corresponds to the values given in these international guidelines, which confirms the correctness of the model and its compliance with international standards.

At the same time, in some studies the effect of correlation is also taken into account, in which case the combined uncertainty is expressed in the following extended form:

$$u_c^2=\sum u_i^2+2\sum u_i u_j r_{ij} \qquad (30)$$

However, in the present work, since the parameters were assumed to be mutually independent, the correlation coefficients were not taken into account. This simplified the calculations and did not have a significant effect on the accuracy of the results [1]. Analysis of the obtained results shows that the evaluation of uncertainty through the modeling method is much more effective than traditional methods, as it allows precise separation of the contribution of each source. This is an important scientific advantage and indicates the expediency of wider application of this approach.

Thus, the results of the present study are consistent with international standards and scientific sources and provide the opportunity for their further development.

The practical application of the developed mathematical model is of great importance in laboratory and industrial fields where analytical balances are used, as it allows increasing the reliability of measurement results and improving metrological support [2]. In particular, in the pharmaceutical, chemical industries and scientific research laboratories, where accurate measurement of micro-masses is required, correct evaluation of uncertainty is a decisive factor. Through this model, the contribution of each uncertainty source is identified, allowing laboratory personnel to determine which factors require special attention, thereby contributing to the optimization of the measurement process.

One of the practically important issues is uncertainty control, which can be expressed by the following relation:

$$u_c \rightarrow \min \qquad (31)$$

that is, minimization of the combined uncertainty is the main objective. To achieve this, it is necessary to reduce the dominant sources, i.e., control of eccentricity and drift is a first-priority task. For example, by accurately determining the geometric center of the balance platform and constantly placing the load at this point, the effect of eccentricity can be significantly reduced. In addition, the effect of drift is reduced by stabilizing the storage conditions of the standard weights and regularly calibrating them.

Another advantage of this approach is that it allows the implementation of a metrological control system in laboratories. Which ensures the reproducibility of measurement results and increases compliance with international standards [3].

Thus, the developed model is an effective tool that can be widely applied in practice. The generalization of the research results shows that the evaluation of uncertainty in the calibration process of analytical balances requires a comprehensive approach, and this approach can be effectively implemented through mathematical modeling [1]. The model developed in this work fully complies with the GUM requirements and allows the measurement result to be considered as a function of all influencing factors. The analysis of the obtained results showed that the main part of the combined uncertainty corresponds to the contribution of systematic sources, in particular, eccentricity and drift of the balance weight manifested themselves as dominant factors. This situation is related to the operating principles of analytical balances and their design features, and its consideration is of great importance for increasing measurement accuracy.

The mathematical approach used in the research made it possible not only to evaluate uncertainty but also to identify opportunities for its reduction. This allows the formation of the following general conclusion:

$$u_c = f(\mathit{environmental\ conditions}, \mathit{analytical\ balance}, \mathit{reference\ standard}) \quad (32)$$

that is, the combined uncertainty is formed as a result of the interaction of several factors. At the same time, the research results show that uncertainty reduction requires a comprehensive impact not on a single factor, but on all dominant sources. This requires a systematic approach in the field of metrology and further increases the practical significance of this work.

Thus, the obtained results contribute to the formation of new scientific and practical approaches to the evaluation and control of uncertainty in the calibration process of analytical balances.

A more detailed analysis of the research results shows that the method of mathematical modeling in the evaluation of uncertainty in analytical balances has high efficiency, as it allows quantitative assessment of the contribution of each source to the measurement result [1]. It is evident from the results obtained in this work that the combined uncertainty is mainly due to the contribution of several dominant sources, and this situation can be generalized by the following relation:

$$u_c^2 \approx u_{ecc}^2 + u_{drift}^2 \qquad (33)$$

where the influence of small sources is considered relatively insignificant. This approximation is very convenient in practice, simplifies calculations, and enables rapid analysis. At the same time, this relation is also of great importance in developing an uncertainty control strategy, since it clearly indicates which sources require primary attention.

The results of the present study also show that, due to the high technical capabilities of analytical balances, random uncertainties are almost negligible, which indicates the advanced level of modern measuring instruments. However, systematic factors, especially environmental conditions and mechanical factors, remain the main limiting factor. Therefore, in future research, it is important to study these factors in greater depth and to develop methods for their reduction [3].

Thus, the model developed in this work can be recommended as an effective and reliable method for evaluating uncertainty in the calibration process of analytical balances.

The generalization of the research results shows that the evaluation of uncertainty in the calibration process of analytical balances is not limited to the calculation of individual factors alone, but requires a comprehensive approach that also takes into account their interaction [1]. The mathematical model developed in this work meets these requirements and allows the measurement result to be expressed as a function of all main sources. The obtained results showed that eccentricity and drift play the main role in the formation of combined uncertainty, which indicates that measurement accuracy can be significantly increased by controlling them.

Based on the modeling results, the evaluation of uncertainty for analytical balances can be expressed in the following simplified form:

$$u_c \approx \sqrt{u_{ecc}^2 + u_{drift}^2} \qquad (34)$$

This expression is convenient for rapid evaluation in practice, as it allows approximate determination of uncertainty while taking into account the dominant sources. At the same time, this approach is also useful for making quick decisions under laboratory conditions, since it does not require excessive calculations. In addition, this model can be applied to loads in different ranges, which allows it to be evaluated as a universal tool.

Thus, the research results propose an effective and practically significant method for evaluating uncertainty in the calibration process of analytical balances, and the introduction of this method into metrological practice is considered expedient [2].

Within the framework of the present study, a mathematical model for evaluating uncertainty in the calibration process of analytical balances was developed and substantiated in accordance with the requirements of GUM (JCGM 100:2008), which ensures its scientific correctness [1]. As a result of the calculations and analyses performed, the main sources of uncertainty in the measurement process were identified and their contribution to the overall result was quantitatively evaluated. In particular, eccentricity and drift of the balance weight were identified as dominant sources, and it was shown that they constitute the main part of the combined uncertainty. This indicates that in practice, primary attention should be paid to these factors in order to reduce uncertainty.

The research results show that the evaluation of uncertainty through the mathematical modeling method is much more effective than traditional methods, as it allows separate analysis of the contribution of each source. This creates the opportunity for a deeper understanding of the measurement process and its optimization. At the same time, the universality of the developed model allows its application to various types of analytical balances, which further increases its practical significance [2].

Thus, the results of the present study make an important contribution to scientific research in the field of metrology and contribute to the formation of new approaches to the evaluation of uncertainty in the calibration process of analytical balances. This will serve as a basis for further development of research in this direction.

Based on the results of the conducted research, a number of practical recommendations for reducing uncertainty in the calibration process of analytical balances were developed, and their implementation in practice makes it possible to significantly increase measurement accuracy [2]. First of all, the use of centering systems on the balance platform is recommended in order to eliminate off-center loading of the load, since according to formula (23) this factor constitutes a large part of the combined uncertainty. Secondly, to reduce the drift of the balance weight, strict control of its storage conditions, maintenance of stable temperature and humidity, and regular recalibration must be carried out. Thirdly, monitoring of microclimate parameters in the laboratory is of great importance in order to minimize the influence of the air environment, which ensures the stability of measurement results.

Calculations according to formula (27) for evaluating the effectiveness of uncertainty reduction show that if the dominant sources are reduced by 30–50%, the combined uncertainty can decrease by 25–40%, which is a result of great practical importance. This further substantiates the practical application of the modeling results and confirms their reliability.

At the same time, based on the results of the present study, it is possible to introduce standardized methods for evaluating and controlling uncertainty in laboratories, which ensures compliance of measurement results with international requirements [3]. Thus, the developed approach is of high importance not only theoretically but also practically, and its wide application in metrological practice is considered expedient.

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