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LOCAL LIMIT THEOREM, ITS APPLICATIONS AND RESULTS

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Abstract: This article provides information about the local limit theorem, its definition, mathematical expression, and its difference from the central limit theorem. In addition, the article discusses some of the results derived from the theorem and its areas of application.

Keywords : Events, experiments, sequence of experiments, probability of an event, local limit theorem **,** central limit theorem

The local limit theorem is one of the most important theorems in probability theory. This theorem helps to determine the fine details of the distribution of sums of random variables. The development of the local limit theorem occupies an important place in the history of probability theory. The initial forms of this theorem were developed by mathematicians in the late 19th and early 20th centuries. It was initially developed on the basis of the law of large numbers and the central limit theorem, and later began to be widely used in solving complex problems in the field of random processes and probability density. The definition and mathematical expression of the theorem are as follows :

According to the classical definition of the local limit theorem: if $X_1, X_2, ..., X_n$ there are mutually independent random variables, which have the same distribution and are of finite variance, then the probability density function of the sum of these random variables approaches the Gaussian (or normal) distribution density for increasing numbers.

In mathematical language, if $S_n = X_1 + X_2 + ... + X_n$ the sum is , then:

$$P(S_n = k) \approx \frac{1}{\sqrt{2\pi n\sigma^2}} \exp\ -\frac{(k-n\mu)^2}{2n\sigma^2}\ , \tag{1.1}$$

where and σ^2 are μ the mean and variance, respectively.

It is important to understand the difference between the local limit theorem and the central limit theorem. According to the central limit theorem, the sum of a large number of independent random variables will converge to a normal distribution, but this convergence will occur at a point in the middle of the general probability distribution. In other words, the central limit theorem considers the convergence of the probability density function. The local limit theorem, on the other hand, deals with the probabilities themselves, that is, it allows us to determine the probability that the sum of random variables will take certain values. This allows us to study the

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finer details of the convergence process, since it does not examine the density of the distribution, but the probabilities corresponding to the values of the variable.

Basic concepts and mathematical basis: To understand the local limit theorem, it is necessary to know some basic concepts, mathematical expressions, and important principles of probability theory.

A random variable is an event whose value can change. For example, a coin toss may result in a "heads" or a "number". Mathematically, a random variable is described by X its possible values and the probabilities that correspond to them. The concept of a probability distribution is used to describe these probabilities.

Probability distributions can be discrete or continuous:

Discrete distribution: A random variable that takes on discrete values, such as whole numbers. Each value has a defined probability, such as in a coin toss. $P\left(X = gerb\right) \;=\; 0.5$

Continuous distribution : A random variable takes on continuous values. In this case, the distribution is defined using a probability density function. For example, a normal distribution.

Basic concepts of mathematical probability theory:

1. Mathematical expectation (mean value): It is defined as the average value of a random variable and $E(X)$ is determined by.

$$E(X) = \sum_{i} x_i P(X = x_i) \tag{1.2}$$

or

$$E(X) = \int_{-} xf(x)dx \tag{1.3}$$

where is $f(x)$ the probability density function.

2. Dispersion: Provides information about how much a random variable deviates from the mean.

$$Var(X) = E[(X - E(X))^2] \tag{1.4}$$

3. Standard deviation: It is the square root of the variance and measures the degree of spread.

$$\sigma = \sqrt{Var(X)} \tag{1.5}$$

The main idea in the proof of the local limit theorem is to estimate the distribution of the sum of random variables by means of the characteristic function. This process requires the use of Fourier transforms and asymptotic methods, since it can be difficult to calculate the distribution exactly when the sum of random variables is large.

Another important challenge is to determine the degree of asymptotic convergence that occurs during the normalization of the sum and the removal of the mean values. This convergence is estimated based on the large number of values.

Several basic mathematical methods are used in the proof of the local limit theorem:

- Fourier analysis: Calculating the probability density using the characteristic function and Fourier inversion plays a key role in proving the local limit theorem.
- Asymptotic analysis: Asymptotic methods are used to evaluate the convergence of a distribution to a normal distribution. This method ensures the accuracy of the proof of the local limit theorem.
- Markov chains and probability chain theory: This theory is used to evaluate the statistical properties of random processes and find their limit distributions.

The mathematical proof of the local limit theorem leads to the following results:

I. When the sum of random variables is large , its distribution approaches the normal distribution. The accuracy of this approximation depends on the variance of the random variables and the conditions of their distribution.

II. Probability density approximation: Using the theorem, the probability density of the sum is accurately estimated and it is approximated by a Gaussian density function.

III. Asymptotic exact formulas: The proof of the local limit theorem yields asymptotic exact formulas for sums of random variables, which are useful in solving many practical problems .

The proof of the local limit theorem explains its mathematical basis and theoretical principles. The proof of this theorem uses Fourier analysis, asymptotic methods, and the theory of large numbers. This theorem allows for a deep analysis of the distribution of the sum of random variables and is widely used in probability theory, statistics, and other mathematical fields.

Extensions and generalizations of the local limit theorem

this section, we will introduce various generalizations and extensions of the local limit theorem. They allow us to apply the theorem to a wider class of random variables and are important for understanding how well the theorem works under different conditions.

The classical local limit theorem is valid for uniformly distributed and independent random variables. However, in many practical problems, the random variables may not have the same distribution or may be correlated. Therefore, it is necessary to adapt the local limit theorem to more general conditions.

- Non -Uniform Distributions: If the random variables do not have a uniform distribution, their means and variances differ. In this case, general forms of the central limit theorem are used to extend the theorem. In this case, the distribution of the sum of the random variables also approaches a normal distribution, but the parameters vary depending on the specific properties of each random variable.

➢ are related , that is, if there is some correlation between them, the local limit theorem is used in a modified form. In these cases, the speed of convergence and the shape of the distribution change depending on the degree of coupling between the random variables.

Local limit theorem for stochastic processes and time series

The local limit theorem can be extended to analyze not only sums of random variables, but also various random processes. For example, there are extensions of the local limit theorem to time series, Markov chains, or other complex random processes .

✓ Markov chains: The connection between states in Markov chains is different from the form of the local limit theorem used for ordinary random variables. In this case, special forms of the local limit theorem are used to study the long-term behavior of the chain.

✓ Time series: If a random process is given as a sequence that varies over time, extended forms of the local limit theorem are used to describe the approximation to a normal distribution over a certain portion of the time series.

Asymptotic theory and convergence at infinity

One extension of the local limit theorem is asymptotic approximation, that is, determining how the distribution of random variables converges at large values. Asymptotic approximation problems help us study the approximation of the normal distribution for an infinite number of random variables. This process takes into account the following factors:

o Large number limits: It is important to study how quickly the theorem converges for very large sums of random variables.

o Dispersion and rate of spread: How quickly a distribution approaches a normal distribution depends on the variance of the random variables and their spread.

Extensions of the local limit theorem significantly expand its scope. The theorem can be generalized to various classes of random variables, thick-tailed distributions, random processes, and time series. These extensions allow the local limit theorem to be applied in more sophisticated situations and are important for a deeper understanding of the behavior of random processes.

Applications and practical examples of the local limit theorem :

The local limit theorem is used in probability theory to model various phenomena. Examples:

❖ Games and Gambling: The local limit theorem can be used to analyze gambling and various game processes. For example, the theorem can be used to model the distribution of outcomes of random games such as coin tossing or dice rolling. In these games, the distribution of the sums obtained from a large number of tosses approaches a normal distribution.

❖ Statistical tests and evaluations: The local limit theorem is used to evaluate the results of random processes in various statistical analyses, such as product quality, experimental results, and social studies. This theorem can be used to determine the probability of the results obtained and to perform statistical tests. The theorem is used to study important processes in areas such as:

Physics and natural sciences:

physics and natural sciences, the local limit theorem is used for many random processes:

1. Random motion of particles (Brownian motion): The local limit theorem is used to model the motion of microscopic particles. The random motions of particles and their positions approach a normal distribution over time.
2. Energy distribution: In molecular physics, the local limit theorem is helpful in analyzing the distribution of kinetic energy of particles. The sum of the energies of particles over a large volume can have a normal distribution .

Financial Markets and Economy:

In financial markets and economics, the local limit theorem is used to model and evaluate various processes:

1. Stock market and price dynamics: The local limit theorem can be used to analyze stock price movements, income distributions, and financial asset values. For example, if the sum of stock price movements approaches a normal distribution, this phenomenon can be modeled using the local limit theorem.
2. Risk Analysis and Insurance: In the insurance industry, the local limit theorem is used to analyze the combined effect of various risk factors. As a result of random risk events, the sum of insurance payments approaches a normal distribution, which helps in calculating insurance premiums.

Social sciences and humanities:

The local limit theorem can also be applied in the social sciences:

1. Public Opinion Polls: In modeling the distribution of results from social research and public opinion polls, the theorem states that the overall distribution of results from a large number of respondents approximates a normal distribution.
2. Random models of human behavior: When social phenomena or human behavior are modeled as random processes, the distribution and probability of these phenomena can be calculated using the local limit theorem.

Engineering and signal processing:

engineering and signal processing:

1. Noise modeling: The local limit theorem is used to analyze the distribution of noise and signal in telecommunication systems. The sum of large random noise components approaches a normal distribution.
2. Digital filtering and signal processing: In engineering, the local limit theorem is used to analyze and model the probability distributions of various signals.

We can be sure of this in the example of these samples:

a) The dice rolling problem: Suppose we roll a die n times and find the sum of the outcomes. Using the local limit theorem, we can determine that the distribution of this sum approaches a normal distribution for large values of n.
b) Stock market price movements: The intraday movements of a stock index can be modeled as a random process. If these movements are independent and uniformly distributed, the local limit theorem can be used to analyze the overall behavior of the stock market.
c) Calculation of insurance premiums: Since insured events are random , the sum of their premiums approaches a normal distribution for a large number of policyholders. This can be used to calculate insurance premiums accurately.

is widely used not only in mathematical theory, but also in many practical areas. The application of the theorem is important in the fields of statistical analysis, physics, financial markets, social sciences , and engineering in modeling the outcomes of random processes and determining their distribution.

Conclusion

The local limit theorem is one of the basic principles of statistical probability theory. It shows its theoretical and practical importance in determining the distribution of the sum of random events in the limit state. The local limit theorem is a basic tool in studying the distribution of the sum of random quantities , which is of great importance in probability theory and its practical applications. It is widely used not only in theoretical probability theory, but also in solving practical problems. Its results serve to strengthen statistical analysis in many areas and to provide a deeper understanding of the behavior of random processes. The theorem proves that the sum of random events approaches the normal distribution under certain conditions, which makes it possible to simplify complex calculations in many practical problems.

include statistics, finance, physics , and computer science. The results of the local limit theorem play an important role in the analysis of large systems and in determining the behavior of random processes. Therefore, it has a relevant scientific basis in theoretical and applied research. The results of the theorem allow us to obtain accurate and reliable results in the analysis of large-scale systems.

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