

METHODOLOGY OF TEACHING THE LAWS OF ELECTROSTATICS BASED ON
MAXWELL'S EQUATIONS

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Annotation

This article presents a systematic analysis of the fundamental laws of electrostatics, including the law of charge conservation, Coulomb's law, electric field intensity, the superposition principle, and the Ostrogradsky–Gauss law, from the standpoint of modern electromagnetic field theory. Electrostatic phenomena are interpreted as a special (stationary) case of Maxwell's equations, allowing their physical nature to be explained within a unified theoretical framework.

Keywords

electrostatics, Maxwell's equations, Gauss's law, Coulomb's law, electric field, superposition principle, physics teaching methodology.

Electrostatics is a branch of physics that studies the interaction of stationary electric charges and the electric field they create. This section explains the fundamental laws of electrical phenomena.

Law of Conservation of Electric Charge

Definition: The algebraic sum of electric charges in a closed system remains constant. Charge is neither created nor destroyed; it only transfers from one body to another(1).

$$\Sigma q = const \quad (1)$$

An analysis of the law of conservation of electric charge shows that electric charge can neither be created nor destroyed; it is only transferred from one body to another.

Coulomb's Law

Definition: The force of interaction between two point charges is directly proportional to the product of their charges and inversely proportional to the square of the distance between them. Like charges repel each other, and unlike charges attract each other(2).

$$F = k \frac{|q_1| \cdot |q_2|}{r^2} \quad (2)$$

$k=9 \cdot 10^9 \text{ N} \cdot \text{m}^2 / \text{C}^2$ - proportionality constant

Charges of the same sign repel one another, while charges of opposite signs attract one another.

Electric Field and Superposition Principle

The medium formed around charges that influences other charges is called the electric field. Its strength is characterized by the electric field intensity, defined as the force acting on a unit positive test charge placed in the field (3).

$$E = \frac{F}{q} \quad (3)$$

The electric field intensity created by a point charge is independent of the magnitude of the test charge placed in the field (4).

$$E = k \frac{|q|}{r^2} \quad (4)$$

If an electric field is generated by multiple charges, the resultant field intensity is found as the vector sum of the fields produced by each charge individually, in accordance with the superposition principle (5).

$$E = E_1 + E_2 + E_3 + \dots + E_n \quad (5)$$

Ostrogradsky–Gauss Law

Definition: The electric flux through a closed surface is proportional to the total charge enclosed within that surface. Gauss’s law is especially useful for solving symmetric electrostatic problems (6).

$$\Phi = \frac{q}{\epsilon_0} \quad (6)$$

Gauss’s law is particularly useful in electrostatics for solving problems that possess geometric symmetry.

The basic laws of electrostatics form the theoretical foundation of electromagnetic theory. In modern physics education, electrostatics serves as the conceptual basis for understanding electromagnetism. Teaching this section through Maxwell’s equations promotes systematic thinking and the formation of a scientific worldview.

Between 1860 and 1865, the English scientist James Clerk Maxwell generalized electrical and magnetic phenomena and proposed the existence of a new form of matter — the electromagnetic field. According to this concept, electric and magnetic fields are two manifestations of a unified electromagnetic field. Maxwell formulated a system of equations describing these processes.

Interpreting electrostatic phenomena as special (stationary) cases of Maxwell’s equations has significant methodological value in education. This approach enables students to view electric and magnetic phenomena not as independent topics but as interconnected parts of a unified theory.

$$\left\{ \begin{array}{l} \text{div} \vec{E} = \frac{\rho}{\epsilon_0} \quad (1) \\ \text{rot} \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (2) \\ \text{div} \vec{B} = 0 \quad (3) \\ \text{rot} \vec{B} = \mu_0 \vec{j} + \mu_0 \epsilon_0 \frac{\partial \vec{E}}{\partial t} \quad (4) \end{array} \right.$$

Treating equations (1) and (2) of Maxwell’s equations as particular (boundary) cases of electrostatic phenomena has important methodological value in physics instruction. This approach allows students to view electric and magnetic phenomena not as isolated sections but as interconnected components of a unified electromagnetic field theory. In electrostatics, these two equations alone encompass all the fundamental laws. The process can be examined step by step.

Charge and Field Concepts.

Electric charge is a fundamental concept of electrodynamics, since a system of charged particles serves as the source of the electromagnetic field. Charge itself is regarded as a distinct, independent physical property that exists separately from, yet is carried by, particles. A particle may exist without charge; for example, a neutron is electrically neutral, that is, it has no charge. However, although charge is considered an independent physical quantity, it cannot exist without a carrier particle. Therefore, electric charge is treated as an inseparable property of particles. The magnitude of charge is determined through the force of interaction between point charges. The electric field is formed around a charge and acts on other charges. In this way, students perceive the field as a real physical medium rather than an abstract concept.

Gauss’s law demonstrates that electric charge acts as the source of the electric field. This relationship is mathematically expressed by equation (1) of Maxwell’s equations.

$$\oint \vec{E} \cdot d\vec{S} = q / \epsilon_0$$

Deriving Coulomb's law from Maxwell's equations, specifically using Gauss's law, is considered a scientifically, logically, and methodically effective approach in teaching electrostatics. Using this method, Coulomb's law is not presented as a ready-made empirical formula, but rather is justified as a specific result derived from the general laws of electromagnetic field theory.

In this approach, first the electrostatic form of Maxwell's equations — Gauss's law — is considered:

$$\oint E \cdot dS = q / \epsilon_0.$$

For a point charge, due to spherical symmetry, the field intensity is the same at the surface of a sphere, and the flux equals $E \cdot 4\pi r^2$.

$$E \cdot 4\pi r^2 = q / \epsilon_0,$$

$$E = \frac{1}{4\pi\epsilon_0} \cdot \frac{q}{r^2}$$

The force acting on a test charge is determined by $F = q_0 E$, and in this way, Coulomb's law is obtained:

$$F = \frac{1}{4\pi\epsilon_0} \cdot \frac{|q| \cdot |q_0|}{r^2}$$

By carrying out this process, students develop the idea of interaction through the field, replacing the old notions of action at a distance. As a result, the electric field is regarded as a fundamental physical concept, and force is interpreted as a consequence of the field. This aligns with scientific logic and reinforces the conceptual foundations of modern physics.

Deriving the principle of superposition from Maxwell's equations has important methodological significance in teaching electrostatics, as it scientifically justifies the linear nature of the electric field and the fact that electromagnetic phenomena have a unified theoretical basis. In this approach, the principle of superposition is not presented as a ready-made rule but is explained as a natural consequence of the mathematical structure of Maxwell's equations. This helps students develop the skill of understanding the origin of formulas, rather than simply memorizing them.

In electrostatics, one of Maxwell's equations — Gauss's law — is written as follows:

$$\nabla \cdot E = \rho / \epsilon_0.$$

This equation shows that the electric field intensity is linearly related to the charge density.

$$E = E_1 + E_2 + E_3.$$

Thus, the resultant field equals the vector sum of the fields produced by individual sources. In this way, the principle of superposition emerges as a direct mathematical property of Maxwell's equations.

Moreover, this approach develops deductive reasoning. The student derives a specific result from the general equation, that is, logically proves the principle of superposition from Maxwell's equations. This process strengthens skills in logical thinking, mathematical modeling, and understanding cause-and-effect relationships.

It is known that electrostatics is the branch of physics that studies stationary (time-invariant) charges and the electric fields they produce. If, in Maxwell's equations, the fields do not change with time, that is, if we take $\partial/\partial t=0$, then the equations simplify and reduce exactly to the fundamental laws of electrostatics — Coulomb's law, Gauss's theorem, the concept of potential field, and the condition $\text{rot } E=0$. Thus, the fact that electrostatics is a special case of the general electromagnetic theory is justified both mathematically and physically.

From this perspective, explaining electrostatic phenomena in teaching based on Maxwell's equations helps students develop the following scientific concepts:

- the unified nature of electric and magnetic fields;
- the description of all electromagnetic phenomena by general laws;
- the recognition of classical electrodynamics as an integrated theoretical system;
- the understanding that specific laws (Coulomb's law, electric field intensity, potential) are consequences of the general equations.

Therefore, teaching electrostatic phenomena as a special case of Maxwell's equations not only deepens the physical content but also helps students form a complete understanding of the unified, coherent, and fundamental theory of the electromagnetic field.

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