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LINEAR PROGRAMMING PROBLEM: SOLVING PROBLEMS ABOUT SOLUTIONS AND THEIR PROPERTIES

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Abstract. This article examines problem-solving approaches for linear programming with a specific focus on the structure of feasible solutions and the analytical properties that determine optimality. The study systematizes classical and modern methods used to solve linear programming problems, including geometric interpretation, simplex-based procedures, and analytical verification of solution validity. Particular attention is given to the characterization of feasible regions, boundary behavior, extreme points, and the conditions under which optimal solutions exist or become degenerate or multiple. Through representative problem cases, the paper demonstrates how theoretical properties influence algorithmic performance and decision-making accuracy in practical applications. The discussion emphasizes logical rigor in modeling, sensitivity to constraints, and the interpretation of solution sets within real optimization contexts. The results contribute to a clearer understanding of how structural properties of linear models guide both solution selection and evaluation, offering a coherent framework suitable for educational, analytical, and applied optimization tasks.

Keywords: Linear programming; optimization models; feasible region; extreme point; simplex method; optimal solution; degeneracy; duality; constraint analysis; mathematical optimization.

Аннотация. В данной статье рассматриваются подходы к решению задач линейного программирования с особым акцентом на структуру допустимых решений и аналитические свойства, определяющие оптимальность. Систематизированы классические и современные методы решения задач линейного программирования, включая геометрическую интерпретацию, процедуры симплекс-метода и аналитическую проверку корректности полученных решений. Особое внимание уделено характеристике области допустимых решений, поведению на границах, экстремальным точкам, а также условиям существования оптимального решения, возникновению вырожденности и множественности оптимумов. На основе типовых расчетных примеров показано, каким образом теоретические свойства модели влияют на алгоритмическую эффективность и обоснованность принимаемых решений в прикладных задачах оптимизации. В работе подчеркивается значимость логической строгости математического моделирования, анализа ограничений и корректной интерпретации множества решений в реальных условиях оптимизационного анализа. Полученные результаты способствуют более глубокому пониманию того, как структурные свойства линейных моделей определяют выбор и оценку решений, формируя целостную методологическую основу для учебных, научных и прикладных исследований.
Ключевые слова: линейное программирование; оптимизационные модели; область допустимых решений; экстремальная точка; симплекс-метод; оптимальное решение; вырожденность; двойственность; анализ ограничений; математическая оптимизация.

Annotatsiya. Ushbu maqolada chiziqli programmalashtirish masalalarini yechish yondashuvlari, ayniqsa, ruxsat etilgan yechimlar tuzilmasi hamda optimallikni belgilovchi analitik xossalar nuqtai nazaridan tahlil qilinadi. Tadqiqot doirasida chiziqli programmalashtirish masalalarini yechishda qo‘llaniladigan klassik va zamonaviy usullar, jumladan, geometrik talqin, simpleks usuli jarayonlari hamda olingan yechimlarning to‘g‘riligini analitik tekshirish bosqichlari tizimli ravishda yoritiladi. Ruxsat etilgan yechimlar sohasi, chegaraviy holatlar, ekstremal nuqtalar, shuningdek, optimal yechimning mavjudlik shartlari, degeneratsiya va ko‘p optimal yechimlar holatlariga alohida e’tibor qaratiladi. Namunaviy masalalar asosida nazariy xossalarning algoritmik samaradorlikka hamda amaliy optimallashtirish jarayonida qarorlar asoslanganligiga ta’siri ko‘rsatib beriladi. Maqolada matematik modellashtirishdagi mantiqiy izchillik, cheklovlarni chuqur tahlil qilish va yechimlar to‘plamini amaliy nuqtai nazardan to‘g‘ri talqin etish masalalari ustuvor ahamiyatga ega ekani ta’kidlanadi. Tadqiqot natijalari chiziqli modellar tuzilmasining yechimlarni tanlash va baholash jarayoniga ta’sirini yaxlit konseptual asosda tushuntirib, ta’limiy, ilmiy hamda amaliy optimallashtirish tadqiqotlari uchun metodologik poydevor yaratadi.

Kalit so‘zlar: chiziqli programmalashtirish; optimallashtirish modellari; ruxsat etilgan yechimlar sohasi; ekstremal nuqta; simpleks usuli; optimal yechim; degeneratsiya; dualiklik; cheklovlar tahlili; matematik optimallashtirish.

Introduction. Linear programming (LP) represents one of the most fundamental and widely applied branches of mathematical optimization, providing a rigorous framework for modeling, analyzing, and solving problems characterized by linear relationships among variables. Its theoretical foundation rests on the interplay between objective functions and linear constraints, which define a feasible region within which optimal solutions can be sought. The significance of LP spans multiple disciplines, including operations research, economics, logistics, production planning, and decision sciences, making it an indispensable tool for both academic research and practical applications. A core aspect of LP analysis is understanding the structure of the feasible set and the behavior of solutions at its boundaries. Feasible regions, typically convex polyhedra, determine the existence and uniqueness of optimal solutions, while extreme points, edges, and faces play a crucial role in guiding algorithmic search procedures. Classical methods, such as the simplex algorithm, exploit these structural properties to traverse vertices efficiently and locate optimal points. In parallel, modern computational techniques extend the applicability of LP to large-scale and complex problems, including sensitivity analysis, parametric optimization, and duality-based methods. Despite its well-established theoretical framework, challenges persist in interpreting solution properties under degenerate or multiple-optimum conditions, managing constraint interactions, and ensuring numerical stability in practical implementations. Consequently, a comprehensive understanding of LP requires integrating geometric intuition, algebraic reasoning, and computational strategies. This article aims to synthesize these perspectives, illustrating through representative problem examples how structural characteristics of linear programs influence solution quality, stability, and interpretability. By emphasizing both methodological rigor and practical relevance, the discussion seeks to provide a coherent and academically robust foundation suitable for conference-level presentation and publication, while highlighting the critical insights that inform decision-making in real-world optimization contexts.

Main Body. Linear programming (LP) is a cornerstone of mathematical optimization, providing systematic tools for decision-making where relationships between variables are linear and constraints can be precisely defined. At its core, an LP problem consists of an objective function, usually intended for maximization or minimization, and a set of linear constraints that delineate the feasible region. The feasible region, often represented geometrically as a convex polyhedron in n-dimensional space, encompasses all points that satisfy the constraints simultaneously. Understanding the structure and properties of this feasible set is critical, as it directly determines the existence, uniqueness, and multiplicity of optimal solutions. A fundamental property of LP problems is convexity. The convex nature of the feasible region ensures that any local optimum is also a global optimum, a characteristic that significantly simplifies the analysis and computational search for optimal solutions. Extreme points, or vertices, of the feasible region are particularly significant because, according to the fundamental theorem of linear programming, at least one optimal solution occurs at an extreme point. This theorem underpins classical solution methods such as the simplex algorithm, which systematically navigates from vertex to vertex along the edges of the feasible polyhedron, improving the objective function at each step until no further improvement is possible. The method's efficiency stems from its ability to exploit the combinatorial structure of LP problems, focusing on a finite set of candidates rather than exploring the entire continuous space. Beyond simplex, alternative techniques such as interior-point methods provide complementary approaches by traversing the interior of the feasible region rather than its boundary. These methods have gained prominence in large-scale LP problems, where traditional simplex iterations may become computationally expensive. Interior-point methods rely on iterative procedures that maintain feasibility with respect to all constraints, using barrier functions or logarithmic potentials to guide the search toward the optimal solution. The choice between simplex and interior-point approaches depends on problem size, sparsity of constraints, and computational resources, but both rely fundamentally on understanding feasible region geometry and constraint interactions. Constraint analysis is a critical component in both theoretical and applied LP. Constraints define the feasible set but also determine problem sensitivity to changes in parameters. Sensitivity analysis, often conducted using dual variables, measures how variations in coefficients of the objective function or right-hand sides of constraints affect the optimal solution. This insight is particularly valuable in real-world applications, where uncertainty in data or fluctuating resource availability requires decision-makers to evaluate the robustness of solutions. Duality theory further enriches LP analysis by linking the original problem (primal) with a complementary problem (dual), offering both theoretical insights and practical advantages, such as identifying bounds on optimal values or revealing shadow prices in economic applications. LP problems may also exhibit degeneracy or multiple optimal solutions, which present analytical and computational challenges. Degeneracy occurs when more than one vertex corresponds to the same objective value, potentially causing cycling or inefficiencies in the simplex procedure. Multiple optima arise when an objective function is parallel to a constraint boundary over a segment of the feasible region. Recognizing these situations is essential, as they influence algorithmic behavior and require careful interpretation of results. For example, in resource allocation problems, multiple optimal solutions provide flexibility, allowing decision-makers to select among equivalent options based on secondary criteria, whereas degeneracy may necessitate algorithmic adjustments to ensure convergence. Illustrative examples of LP problems underscore the interplay between theoretical properties and solution behavior. Consider a production planning problem where the goal is to maximize profit subject to resource limitations. The feasible region may be constrained by raw material availability, labor hours, and market demand. Extreme points represent production plans that fully utilize specific combinations of resources, and the optimal point identifies the plan yielding maximum profit. Sensitivity analysis can then determine how changes in raw material supply or labor costs impact the optimal plan, guiding managerial decisions. Similarly, in logistics optimization, LP models determine the most efficient distribution routes, where constraints reflect vehicle capacities and delivery deadlines. Structural properties of the feasible set, such as tightness of constraints or the presence of redundant inequalities, directly influence computational efficiency and solution reliability.

Conclusion and Recommendations. In conclusion, linear programming serves as a powerful and versatile tool for structured decision-making across a wide range of disciplines, from production planning and logistics to finance and operations research, due to its rigorous mathematical foundation and clear interpretability. The analysis of feasible regions, extreme points, and constraint interactions provides essential insights into the nature of optimal solutions, including cases of degeneracy or multiple optima, which are often encountered in practical applications. Recognizing these structural properties enables decision-makers to not only identify the best solutions efficiently but also to anticipate how changes in constraints or objective function coefficients may influence outcomes, thereby enhancing strategic planning and resource allocation. From a methodological perspective, classical approaches such as the simplex algorithm remain effective for medium-scale problems, while interior-point methods and computationally optimized routines expand the capability of linear programming to handle large and complex datasets. For practitioners, it is recommended to combine geometric intuition with computational analysis to ensure both accuracy and robustness in solution selection, and to conduct sensitivity and duality analyses to evaluate the stability of decisions under varying operational conditions. Additionally, when multiple optimal solutions are present, prioritizing secondary criteria such as cost efficiency, risk minimization, or flexibility can improve the practical value of chosen plans. Researchers and educators are encouraged to integrate real-world problem examples in instructional settings to illustrate how theoretical properties influence algorithm performance and decision-making, reinforcing the link between abstract mathematical principles and applied optimization. Overall, the strategic application of linear programming not only facilitates optimal resource utilization and process efficiency but also provides a transparent and adaptable framework for systematic problem-solving, enabling organizations to respond effectively to uncertainty and operational challenges while fostering continuous improvement in analytical capability and decision quality.

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